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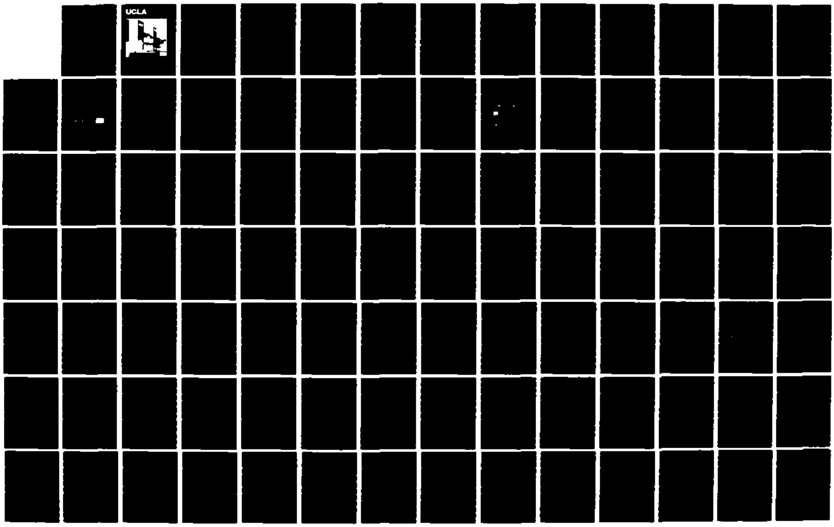

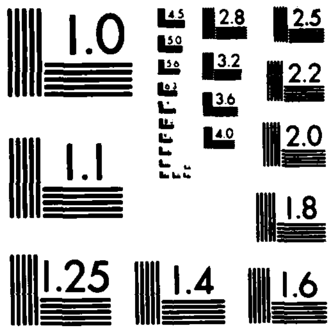

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ON CHANNEL SHARING IN DISCRETE-TIME, MULTI-ACCESS BROADCAST COMMUNICATIONS

Contract MDA-903-77-C-0272

Yechiam Yemini

September 1980
Report No. CSD-830606

COMPUTER SCIENCE DEPARTMENT OFFICERS

CENTER FOR EXPERIMENTAL COMPUTER SCIENCE

MANUFACTURING ENGINEERING PROGRAM

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UCLA-ENG-8227
SEPTEMBER 1980

ON CHANNEL SHARING IN DISCRETE-TIME, MULTI-ACCESS BROADCAST COMMUNICATION

by
Yechiam Yemini

This research, conducted under the chairmanship of Professor Leonard Kleinrock, was sponsored by the Defense Advanced Research Projects Agency, Department of Defense.

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A

ABSTRACT

The major contribution ~~of this dissertation~~ is the ~~introduction and application of~~ novel techniques to solve a variety of distributed resource sharing problems arising in Packet Radio Networks (PRNETs). Some ~~of these transcend the immediate setting in which they are introduced and would be~~ are applicable to a large class of resource sharing problems in computer communication networks. The results ~~obtained fall into three categories:~~ are

(1) *Problems of adaptive channel sharing algorithms:* There are two major contributions falling under this category:

(a) A novel distributed adaptive channel-access scheme, *the Urn scheme,* has been derived mathematically. The Urn scheme adapts to the channel traffic, performing similar to ALOHA for light traffic and converging smoothly to Time Division Multiple Access for heavy traffic, eliminating collisions and exploiting the full channel capacity; in the medium range, it outperforms both schemes. The Urn scheme is proved to be optimal among a large class of access schemes and it lends itself to a variety of robust distributed implementations, thus offering a practical alternative to classical schemes.

(b) A novel mathematical approach to decentralized optimal resource sharing is developed. Using this approach, a very general characterization of optimal distributed access schemes for multi-hop networks is derived. For a single-hop network the optimalty rules implies such diverse access schemes as ALOHA, the Urn scheme and perfect-scheduling. For a multi-hop network, a terra-incognita, the rule implies a variety of novel, relaxation-type, decentralized, optimal access schemes.

2. *Problems of interfering queueing processes:* The problems of interfering queueing processes arise in computer communication networks quite naturally. Queueing processes may interfere with each other through their arrival processes (e.g., "join the shortest queue" routing) or through their service processes (e.g., destructive collisions in PRNETs). We develop novel analytical solutions, exact and approximate, to problems of interfering queues in PRNETs. We introduce a generalization of the Wiener-Hopf factorization technique to solve some general interfering pairs of queueing processes.

3) *Capacity of multi-hop networks:* We compute the capacity of tandems and show that in the limit, when the length of the tandem increases to infinity, the capacity converges to 4/27 of the bandwidth. A novel phenomena of singular topologies (i.e., where topology helps reduce interference) in PRNETs is explored.

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1. INTRODUCTION

1.1 SOME READING SUGGESTIONS

Most sections of the introduction may be skipped by a reader familiar with computer communication networks. I have tried to make each chapter as self contained as possible. The unavoidable price is paid in overlaps and repetitions. The best path to pursue in reading this dissertation is to start directly with chapter two and use the introduction as a reference, only when the terminology becomes unclear.

1.2 PACKET RADIO NETWORKS FOR COMPUTER COMMUNICATION

1.2.1 WHAT IS A PACKET RADIO NETWORK

A Packet Radio Unit (PRU) is a digital transceiver which can generate, receive and transmit packets of digital data, over a broadcast channel. A PRU possesses some limited intelligence which enables it to make simple decisions, a limited buffering facility which enables it to store packets, and a limited range of transmission and reception, which enables it to form a community together with fellow PRUs.

A Packet Radio Network (PRNET) is a community of PRUs [KAHN75, KAHN77, KAHN78, KLEI76, ROBE72, FRAN75, BURC75]. Community members may wish to talk to each other at unpredictable times. When a direct communication is impossible (because of a limited range or physical barriers) the PRUs may employ the network as a relay mechanism to store-and-forward

their packets to the destinations. Some PRUs may be distinguished as *terminals*, i.e., packet producers, some function as *repeaters* to relay packets towards their destinations, and some function as *stations* which possess special processing capabilities. In this case, packets are generated at the terminals and relayed by the repeaters until they reach the station which provides the terminals with services. Figure 1.2-1 illustrates the typical elements of a PRNET.

Each PRU possesses a buffer where packets are stored until they are delivered. We shall usually assume the buffer to be infinite. To develop markovian models of the queueing processes in a PRNET it is necessary to consider the state of each PRU as described by the total number stored in its buffer. Many problems, however, may be solved in terms of a simpler (but non-markovian) state description.

A PRU having a packet ready for transmission is said to be *busy* (also occupied, ready etc...); a PRU which is not busy is said to be idle (also empty, unoccupied). The *state of occupancy* (business) of a PRNET is a description of the business status of each member, in terms of a vector whose i-th coordinate assumes the value 1 iff the i-th PRU is busy, and 0 otherwise.

Usually the major objective of the PRNET community is to deliver packets to destinations, with high reliability in a minimum time.

Let us examine the essential features characterizing PRNETs:

1.2.1.1 Communication channel

The problem of sharing the communication channel is the major problem with which we shall be concerned. Let us describe what "channel" means

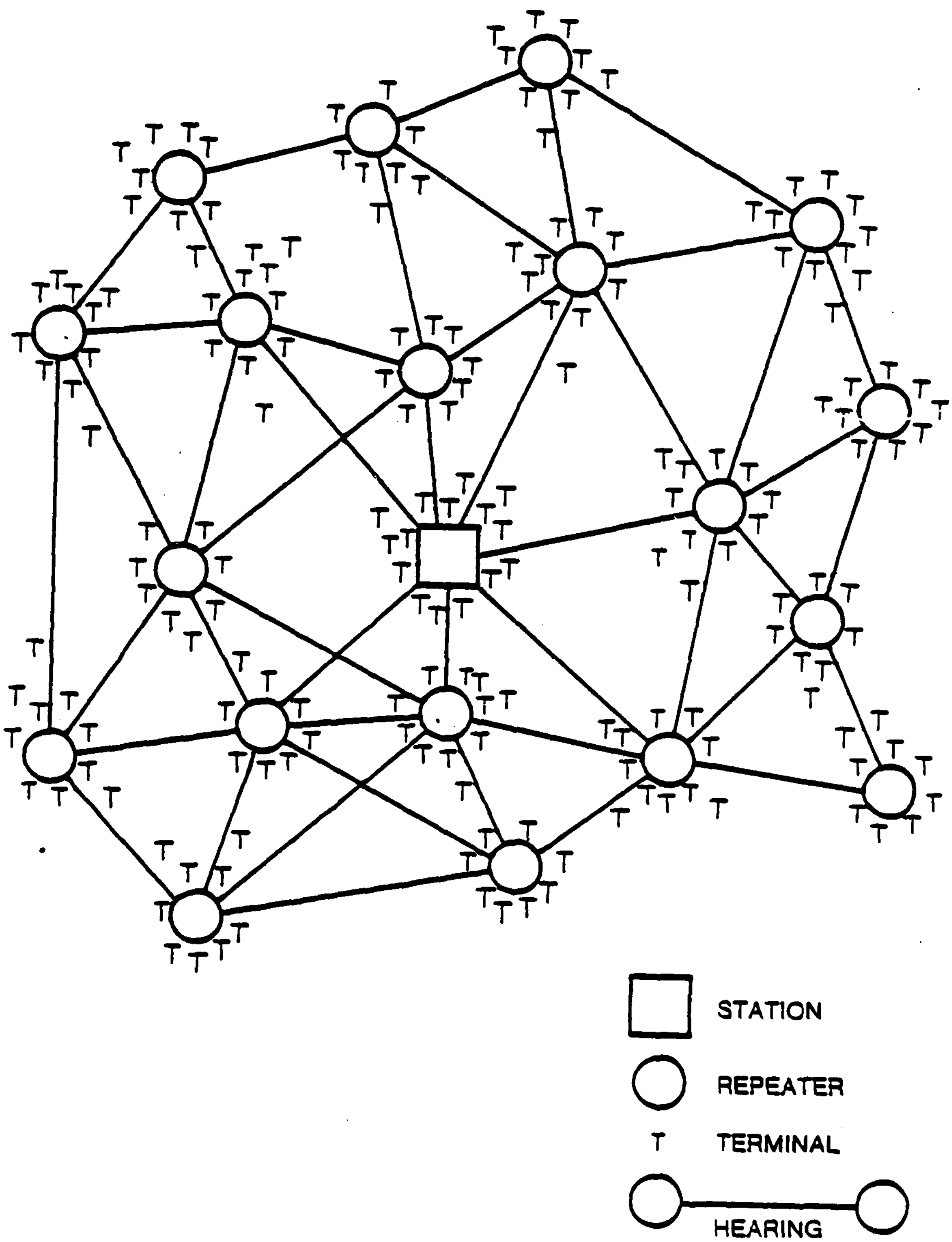

Figure 1.2-1: Typical Structure of a PRNET

throughout this dissertation.

The communication medium is usually a radio channel. However some PRNETs (in the broad sense defined above) may use wire communication [METC76]. In fact, ANY medium which can support a multi-access broadcast channel will do.

Our point of departure is precisely that at which the regime of Communication Theory ends. That is, we assume that problems of modulation, coding, synchronization, and the like, have all been solved one way or another. The channel appears to us as a band stretching in time to infinity.

The channel can be slotted time-wise or frequency-wise. That is, members of the PRNET can identify portions of the channel (thus decide upon their ownership). The models that are considered in this work use *time slotted* channels. The important feature of time slots is that they are recognized by the PRUs as the shareable portions of the communication resource. Slots are usually of a uniform size, that which is required to deliver a packet. Thus the channel appears as a succession of rectangular slots. We shall make the assumption that slotting causes only negligible loss of the channel resource. This is a good approximation as long as the slots are not too thin relative to the synchronization time and the maximal propagation delay. Time slotting defines a global reference system through which users can reach some coordinated channel usage. First it enables users to reach some agreement about channel allocation. Second, it reduces the periods of channel waste by interfering transmissions to half their size for unslotted channel [ABRA73].

we further assume that the three types of channel events are possible; "successful" transmission slot, i.e., a single PRU has been using the slot; an "unused" slot, i.e., no PRU has been transmitting; a "collision", i.e., two or more PRUs have made an attempt to use the same slot. Collisions are assumed to be destructive, i.e., no packet is delivered by a collision slot. We shall use the word "idle" to distinguish those slots which are unused because the system is idle, from slots which are unused when the system is busy. The later type of slots, to be called "empty" are a form of channel waste, while the former represent normal idleness of the service mechanism. Figure 1.2-2 depicts our model for the channel and possible channel events.

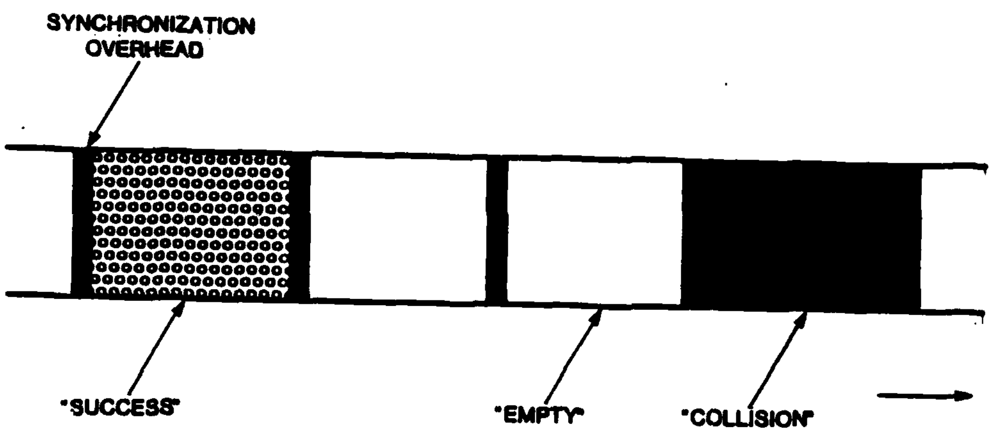

Figure 1.2-2: Generic Channel Events

The channel, as viewed in this dissertation, is merely a spatially distributed resource which is to be shared by the PRNET community. The problem is to

design and analyze sharing algorithms. This places the problems (and solutions) beyond the scope of communication theories. Indeed, the problems that we deal with are typical to any real-time, decentralized, demand-allocation resource sharing systems. The sharing of other computer network resources such as distributed memory systems, distributed processing units or, for that matter, any spatially distributed resources pose similar problems. See [LESS78] for typical examples of real-time distributed processing as well as further references.

Simulation and analytic studies show -- for instance [TOBA77] -- that the communication channel is the critical network resource. That is, small changes in the amount of channel available or its allocation policy, influence the performance significantly. On the other hand, changes in traditional communication resources such as buffers, have negligible effects upon performance (a buffer for two packets in each PRU or for an infinite number of packets, hardly makes a difference [TOBA77]). This is the first major feature in which PRNETs differ from classical packet-switching networks.

1.2.1.2 Hearing topology

Two PRUs which hear each other are said to be in a *hearing* relation. Hearing is a binary relation which we assume to be symmetric. The assumption of symmetry can be easily removed to get more accurate models; it is used for a simplification of the arguments and notation.

The hearing relation may be represented in terms of a hearing graph which characterizes the spatial distribution of the channel resource. The hearing graph is subject to a random time evolution. However, we make the assumption

that the speed at which the hearing topology changes is very small w.r.t. packet transmission time. That is we assume that the hearing topology is essentially static.

1.2.1.3 Communication traffic demand.

The arrivals of packets to the network are subject to random laws. We assume that the communication demand is *bursty*. That is, PRU's channel needs are irregular, infrequent and restricted to small time intervals. Computer communication traffic is typically very bursty. Burstiness is a key notion to the understanding of PRNETs design. A possible definition of a measure for burstiness has been recently proposed [AKAV78, LAM78]. Loosely speaking, burstiness may be characterized through moments of the arrival processes. A first order condition is that the overall traffic generated over an average packet delivery time should be small. A second order condition should reflect the time irregularity of local arrival processes. A possible measure is the spatial average of the variances of packet inter arrival times; for burstiness the variance should be large.

We model the arrival process at each PRU as a Bernoulli process whereby, at each time slot, nature (PRNET users) tosses biased coins and decides whether to generate a packet at each PRU or not accordingly. Independent, spatially distributed Bernoulli arrivals serve as a reasonable model of burstiness.

1.2.1.4 Objective

The objective of the PRNET is to make an efficient use of the channel so as to minimize the overall expected packet delay.

In the subsequent chapters we consider some typical problems of modeling

and analysis of PRNETs. The study of PRNETs is in essence a study of distributed resource sharing. As such, it has far reaching implications to other problems of computer communication. The solutions to the problems of PRNETs have already stimulated new ideas in such, seemingly unrelated, fields such as memory organization. With the advent of increasingly cheaper communication and processing technologies, it is expected that many more problems of distributed resource sharing will have to be solved by computer networks (e.g., adaptive highway traffic-control, adaptive routing of a fleet of vehicles, adaptive sharing of information processing resources). Although this work is concerned with problems of PRNETs, some methods and results (in particular chapter 2, 4 and 5) transcend the immediate setting and could be employed to attack other problems of spatially distributed resource sharing. Work towards this goal will be carried in the future.

1.2.2 SOCIAL ORGANIZATION OF PRNETS

The set-of rules that governs the usage of the communication resources by the PRNET community, is called *communication protocol*. We shall be interested only in the laws that regulate and coordinate the usage of the communication channel; this being the critical resource. There is some similarity between the problem of channel sharing and that of sharing other rare natural resources. For instance, if transmissions are not coordinated the channel may be polluted with "collisions". On the other hand, any scheme for a dynamic coordination of the channel usage must use the very channel to transfer control information. In extreme cases the channel may be completely wasted by the control bureaucracy. If a non-adaptive sharing policy is selected the channel will be underutilized with many "empty" slots which have been reserved for dormant users. Pollution, bureaucracy, underutilization and the like, are typical problems faced by the PRNET community. In what follows we shall describe some of these problems in more detail.

1. *The problem of access schemes*

   The algorithm through which a PRU decides whether he has a right to transmit a packet or not, during a given slot, is called the *access scheme*, [ABRA73, ABRA73a, KLEI76, ROBE73, TOBA75, SCHO76, HAYE77].

2. The decision about access rights is distributed among the community members. If the access scheme is to be adaptive, the decisions must be made in real time (i.e., time between decisions is of the same order of magnitude as transmission slots). The problem

of access scheme is thus a typical problem of a real time distributed decision algorithm.

3. *The problem of routing*

Routing is the mechanism to decide which packet goes where, at any instant of time. Routing, again, is a typical problem of real time distributed processing.

A few routing algorithms for packet switching cable networks have been explored and implemented [GERL72, GALL77, McQU78]. Yet, the problem seems to be far from possessing a complete solution. For instance, consider an adaptive routing algorithm where network members exchange routing data in order to be able to adapt. When the network is heavily loaded, should the rate of routing data exchange, increase or decrease? (Heavy traffic is when fast adaptivity is required; it is desired that status data be updated rapidly. However, heavy traffic is when the communication resources are critically required; it is desired that they should not be loaded with control overhead.)

4. *Capacity*

The distributivity of both state observations and decisions, restricts the ability of the network to cope with the decision problem. Some channel and some time will be wasted to transfer

control and coordination data. Another form of time and channel waste occurs because the decisions taken by the network members are not optimal [KLEI77]. The problem of characterizing the limits on the network ability to deliver traffic, is the problem of *capacity*.

The problem is twofold. First, which fraction of the channel capacity (which is available in the sense of information theory) can be actually used? Second, which allocation control policies obtain the capacity bound?

5. *Analysis of delay-throughput performance*

From the point of view of queueing theory, PRNETs are but a large network of interacting queueing processes. Unfortunately the interaction between the different service mechanisms, through the shared channel, poses a difficult queueing problem. The queueing processes can no longer be assumed to be independent. The arrivals to one queue depend upon its service process, as well as service processes at neighboring queues. In addition work is not conserved but lost through collisions. The interaction can not be eliminated through some simple Jackson-like queueing networks [JACK57]. On the contrary, interaction becomes the essential feature to be captured by any reasonable model.

Thus we face a queueing problem which is an order of magnitude more difficult than queueing problems whose solution we know.

Let us consider a concrete example of a PRNET to demonstrate the above issues. Figure 1.2-3 depicts a business configuration and a sequence of channel events for a typical one-hop PRNET.

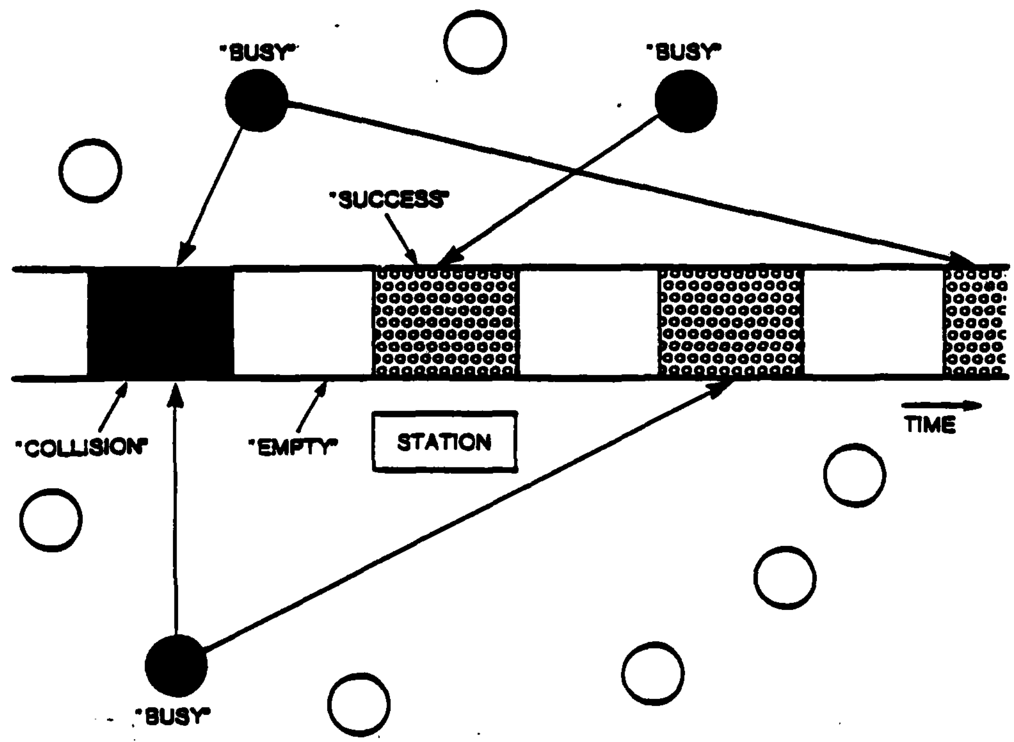

Figure 1.2-3: Typical Business Configuration

The first problem which the PRUs need to decide is: who should have access rights? If they had a master control mechanism to direct them, one busy PRU would have been directed to transmit and the others would remain silent. In

the absence of a centralized control, if each PRU is aware of the precise occupancy configuration, then it is possible to employ a predetermined priority mechanism to resolve the conflict of access right demands over the slot. Unfortunately, the assumptions which we have just made are ideal. In practice no PRU can observe the business configuration. Therefore a well orchestrated decision of access rights can only be achieved at the price of time and channel overhead required for coordination process.

What if each PRU knows nothing at all about the needs of his colleagues? In this case, to prevent an infinite succession of collisions over the channel, it is required to reserve some portion of the channel to each PRU according to a predetermined set of rules. One such rule is round-robin Time Division Multiple Access (TDMA), i.e., round-robin possession of slots. If the traffic is heavy, this rule provides an excellent solution. However, if the traffic is bursty most of the channel is wasted in silence and queueing delays are unreasonably high.

Another solution to the problem of no information is to randomized the decision between transmit or not. Each PRU is equipped with a biased coin; when he is busy, he tosses the coin and decides whether to transmit or not, accordingly. This scheme is a version of the Slotted-ALOHA policy [ABRA73]. Randomization is employed to render lengthy successions of collisions unlikely. There is still an implicit decision problem; namely, how to assign transmission probabilities to the coins. In the absence of any further information it is required that the transmission probabilities be as small as 1/N (N is the total number of PRUs), to be able to cope with the worst case of a fully loaded system.

Unfortunately the expected delay is large and channel slots are wasted in both collisions and empty slots. Thus it is necessary to employ further information, if the system is to be efficient. The transmission probabilities should be adapted to the state of the system. The need to adapt requires a control mechanism to exchange information and coordinate the distributed decisions, leading to a whole class of controlled ALOHA policies [LAM74, FERG75, CARL75, FAYO77, GERL77].

Between no information and perfect information, a spectrum of information structures exists. It is possible to consider an intermediate information structure and search for the optimal adaptive policy to decide access rights on the basis of this information. For instance, suppose the PRUs are aware of the number of busy PRUs but not of their identity. What then is the best decision rule? This particular information (i.e., the number of busy PRUs) is assumed by some controlled ALOHA policies. However, as we shall see, the optimal ALOHA policy is not the best decision rule, given that information. In fact we shall derive the optimal solution and use it to develop a new class of working decision algorithms. Our algorithms use estimates of the total load to decide access rights and exhibit smooth adaptivity to the traffic load.

Next comes the problem of capacity. Suppose we develop an algorithm to decide access rights. Then how much traffic can the network handle? To answer the question we can imagine an experiment where the level of traffic is increased gradually, following some distribution of packet arrivals. At some input rate the queues in the network can no longer be emptied by the service mechanism. The queueing process turns unstable. A reasonable notion of

capacity is provided by the threshold input rate at which the network can no longer sustain the traffic. Thus, the capacity problem is directly related to the stability problem of the network queueing process. At what traffic level does the queueing process become unstable?

Another way of looking at the capacity problem is to consider the network under a heavy traffic, i.e., when all PRUs are constantly busy. How much traffic can the network service? The two views may produce two different notions of capacity, as we shall see. This, seemingly paradoxical result follows from the fact that there may be many paths by which the network may choose to become unstable. Sometimes the optimal path to instability will turn only one queue heavily loaded, while the rest are empty most of the time. Therefore it is not necessary for the network to satisfy the heavy traffic assumption when it turns unstable.

The most important performance measure for a network is its delay-throughput behavior. What is the expected delay as a function of the throughput? This question as well as the following questions are the subject of analysis of the queueing processes in the network. How long are busy periods? What is the effect of changing buffer sizes or their allocation?

Consider the queueing process in the buffer of a generic network member. The access scheme is merely a service process which enables the PRUs to share the server (channel). However, through collisions, the different queues interact and lose service time. The network queueing process is not work conserving; the amount of work which is required to serve a packet depends upon the state of the queues. In fact, the whole idea of channel sharing is to

employ the channel according to the service needs of the users. The analysis of the delay-throughput performance of even simple access scheme models poses very difficult mathematical problems. It is possible to simplify the models to a point where interaction between the queues is completely ignored. However, when we seek to understand the effects of collisions on the performance, we have to face the problem of interaction.

1.3 SCOPE OF THIS WORK

1.3.1 THE APPROACH

PRNETs are large scale systems whose behavior is determined by numerous parameters. The problem of modeling and analyzing the performance of the communication protocol resembles the many-body problem of physics. That is, we try to solve for the behavior of the system from the behavior of its atomic components. However, whereas physicists are interested in description we are in an urgent need of prescriptions. Particles have a built in interaction protocol to minimize the total energy of a physical system; PRUs are yet to be endowed with a communication protocol to minimize the overall expected delay.

Problems of modeling, analysis and optimization of decentralized decision mechanisms seem to have no satisfactory solutions yet (see the recent special publication of the *IEEE Transactions on Automatic Control*, Vol. AC-23, number 2, April 1978, dedicated to these problems). To quote the conclusions of the major survey article [SAND78] (ibid.):

> "Our most fundamental conclusion, after surveying a vast amount of literature, is that.... the question of what structures are desirable for control of large scale systems has not been addressed in a truly scientific fashion. In our opinion, we do not believe that the existing mathematical tools ....are powerful enough to define a preferable structure for decentralized and/or hierarchical control. First, we do not believe that it is reasonable to seek a single best optimal structure. Rather, any future methodologies should strive to define sets of distributed information and control structures that are preferable to others. Second, a unified theory of decentralized control should explicitly include not only traditional index of performance... but in addition
>
> a) the cost of communications
>
> b) reliability issues

c) the cost of computer interfaces

d) the value of incomplete and/or delayed information

e) a formal measure of system complexity.

To develop such a desirable methodology we may have to develop different notions of optimality, principles of optimality and notions of optimal solutions."

A difficult problem of choice confronts the researcher. On one hand it is required to deliver practical solutions for the problems. On the other hand our ability to analyze the performance of a communication protocol is very limited. A compromise is a necessity. We feel that no better words could describe the underlying convictions that lead to our choice of a compromise, than the above quotation. Few additional words are required to describe our specific choice of compromise.

First, we have tried to choose models which are as simple as possible to capture the features of interest. This choice reflects our conviction that at this stage of knowledge, principles and approaches should be developed rather than complex insoluble models. We have tried to make the models transparent for the clarity, applicability, and elegance of a solution is an inverse function of its complexity. Second, in presenting the problems, assumptions and solutions, we have tried to expose all sides of the problem not only those captured by a particular set of assumptions. The solutions are supplemented by discussions of practical implementation, limitations and open problems. This reflects our belief that at this stage of knowledge any attempt to present a simplistic solution to one aspect of the problem, which ignores other intrinsic problems is misleading. Third, most of the solutions were first derived mathematically and

only then understood on an intuitive basis. In presentation we prefered a reverse order. We have tried to avoid a "theorem-proof" approach for we believe that Hilbert's plan to represent mathematics as a collection of formal objects, was not meant to include problems of engineering. An exception is chapter four; the problem of interacting queueing processes seems to require a complex mathematical machinery which we could only reduce to a set of drawings. While this reduction is a great simplification of the problem, further study is required for the solution to be fully understood ("engineeringly speaking").

To summarize, the approach we have chosen is to expose the full scope of the problems and point out alternative methods of attack, as well as difficulties which can not be properly addressed by existing methodologies.

1.3.2 ADAPTIVE ACCESS SCHEMES

The problem of adaptive access schemes is considered in chapter two. Our point of departure is the information which is used for decision of access rights. We characterize some typical categories of information structures used for distributed decisions. Then we assume a particular setting where each PRU is aware of the total number of "busies" and uses this information only, to determine the access right. This we call symmetric homogeneous information; it is more than no information (much more as far as ability to adapt goes) and less than perfect information (much less as far as control overhead is concerned); also, some versions of controlled Slotted ALOHA require precisely this information. Under this assumption we develop an optimal stationary access policy.

We call our scheme: the *Urn scheme*, for, as far as each PRU is concerned, the system looks like an urn with black (for busy) and white (for idle) balls, from which a number of balls is to be drawn to maximize the probability that the sample contains precisely one black ball. The urn scheme performs better than optimally controlled slotted ALOHA and better than TDMA. Under light traffic it performs similarly to ALOHA; as the traffic increases it converges smoothly to TDMA. In particular, there is no limit on the useful capacity of the channel; when the load increases "collisions" and "empties" are virtually eliminated and the full capacity is being used.

The ideal urn scheme may be approximated by a number of practical implementations. We show how symmetric information may be practically acquired at a minimum cost in terms of control overhead (i.e., the amount of

channel control overhead does not depend upon the size or the topology of the PRNET, it is fixed). Urn schemes permit collisions and errors on the part of the decision makers. They are robust vis-a-vis both type of problems, errors in estimating the number of busies and errors resulting from unconcerted distributed decisions.

The performance of the urn scheme has been both analyzed and measured from simulation. We examine the results, comparing the performance of Urn schemes to that of TDMA, optimally controlled Slotted-ALOHA and Perfect scheduling (lower performance bound).

1.3.3 INTERACTING QUEUEING PROCESSES

The problem of interfering queueing processes is the subject of the third and the fourth chapters. We consider the problem in a limited setting of two interacting PRUs. However, the considerations are general enough to serve as a basis for solving general problems of interacting queues in discrete time. (it is possible to generalize the methods that we develop, to solve the general discrete time G/G/2 problem. This will represent a major contribution to queueing theory and is particularly significant in the context of interacting queues in computer networks.) Work towards this goal will be carried in the future.

In chapter three we consider approximate methods to solve four different interaction models of two buffered PRUs. The models are ordered by the level of interference between the arrival and the transmission processes at the two PRUs. The fourth model represents a "maximal interference" model (i.e., the two transmission processes interfere with each other and with the two arrival processes, which again interfere with each other) for which an exact solution is derived. The solution extends to a maximum interference model for any number of PRUs. The maximum interference model possesses some interesting singular features. Namely, it is possible for the network to choose a transmission policy which obtains perfect scheduling, because of the structure of the hearing topology.

The main finding of chapter 3 are: first, that simple mathematical approximations which eliminate the dependencies between the queueing processes (so that they may be solved as a collection of one-dimensional queueing problems) do not provide good results. This category includes

heavy-traffic, low-traffic and diffusion approximations. Moreover, we do not possess any mathematical method to determine the domain of applicability of the approximations. Therefore, a suitable mathematical approximation can only be developed by examining the exact solution of the two-dimensional problem.

Second, physical approximations (i.e., those which are obtained by solving for *perturbed interference topologies*) can provide an excellent approximation to the delay-throughput analysis. The topology of interference may be partially ordered by "increased interference" between PRUs. for the case of two buffered PRUs, beyond a certain "interference threshold" the delay-throughput performance curves were identical (i.e., the delay-throughput performance is not sensitive to perturbations of the topology of interference). Therefore, since the extreme model of maximum interference can be solved exactly, its solution can serve as an excellent approximation to other models which experience less interference. The existence of threshold behavior for larger system will be explored in the future.

We use Kingman's algebraic representation of queueing theory [KING63] to show that the two dimensional queueing problem can not be solved using the methods of classical queueing theory. A new approach is required.

In chapter four we develop new mathematical tools to solve the problem of interaction. The method is essentially a Wiener-Hopf technique over some general Riemann surfaces. The connection between problems of boundary value problems of classical physics and boundary problems of random walks (which is what queueing theory is all about) is the basis for the solution of one dimensional queueing problems. In two dimensions the problem becomes much

more difficult and it is required to employ tools of algebraic geometry to solve it.

True, from a theoretical point of view the problem is solved. However, from a practical point of view the solution algorithm is too complex to generate important information about the behavior the system. Nevertheless the road is now paved to develop a systematic method of approximating the solution under some asymptotic conditions such as heavy traffic. Further work in this direction is in progress.

1.3.4 CAPACITY PROBLEMS

The fifth chapter is concerned with problems of capacity. We consider the capacity problem for a tandem. The first problem is to define the notion of capacity precisely. If we choose to define the capacity as the threshold input rate at which the queueing processes turn unstable, we get a definition which agrees with our needs, but which is not too useful as far as actual computation is concerned. For, unfortunately, very little is known about the stability of multi-dimensional Markovian processes.

Another approach is to try and define some special notions of capacity, based upon the expected behavior of the network as the load grows. A typical assumption is the heavy-traffic assumption. Namely, we wish to compute the stability threshold when all queues are kept busy all of the time. The heavy traffic assumption reduces the capacity problem to that of solving nonlinear recurrence relations.

We solve the recurrence relations for the tandem through a linearization trick. We compute the heavy traffic capacity of a tandem as a function of its length and its asymptotic behavior as the tandem becomes infinitely long. The solution process may be easily applied to tandems which are connected to form more interesting network structures.

Next, we find that the heavy traffic capacity is indeed a lower bound on the capacity of the tandem. The actual capacity is about twice as much, and may be achieved through a "rude" behavior where each PRU transmits with probability one when it has a packet. The ability of the network to sustain such a policy, without getting into an infinite succession of collisions and complete blocking,

is a direct result of the singular structure of the tandem hearing topology. Other singular hearing topologies are presented.

Another approach to the problem of capacity is to examine policies which optimize throughput. This approach views the capacity bound as that which results from the restriction of the class of available policies. Therefore the results which may be obtained can be expected to be of a more general nature. Our idea is to replace the notion of a centralized objective with that of a decentralized objective. We employ ideas taken from mathematical economics to define a notion of "decentralized optimality". We obtain necessary conditions for a policy to be optimal, in the form of a "rule-of-thumb". We show that the optimality conditions include Abramson's characterization of optimal Slotted-ALOHA transmission policies [ABRA73], as a particular instance of the rule when the traffic is heavy. In particular, our conditions yield identical heavy-traffic capacity results for one-hop networks. Moreover, the optimality rule also characterizes the optimal Urn-scheme and the optimal (rude) tandem behavior. Therefore, the rule has a wide range of applicability.

The optimal decentralized policies are characterized in terms of Lagrangian multipliers which represent the global "value" of a successful slot usage by network members, each member having a pre-assigned value. If a global coordination scheme is being used to coordinate the values, then together with the local optimality rule an effective hierarchical access control algorithm may be implemented whereby local decisions require only the information contained in the acknowledgments and the "values", to decide their access rights. It is shown that the necessary conditions characterize both optimally controlled

ALOHA and the optimal Urn scheme. Thus both mechanisms may be implemented in terms of a unified rule of behavior. This striking generality will be explored in future work.

1.4 FUTURE RESEARCH

1.4.1 ADAPTIVE ACCESS SCHEMES

The problem of designing practical adaptive access schemes for a multi-hop network needs to be further explored. In the absence of a suitable decentralized decision theory, we took a practical approach towards the problem. Other solutions should be developed before a suitable theoretical basis may be established. In particular, one would like to have a quantitative model of "real-time" decisions, the class of all decentralized strategies, the process of decomposing the decision algorithm into hierarchical decision environments and the process of information exchange and coordination of decisions.

From a theoretical point of view, one would like to have a suitable theory of team decisions. Computer networks are already turning from yet "just packet-switching mechanisms" into decentralized processing mechanisms, which employ the distributivity of the communication and processing to achieve a common goal. For example, networks of distributed sensors which will employ the communication and processing capabilities, to integrate sensors observations into a distributed tracking algorithm, are currently under study [CMU78]. Such networks may be considered as communities of intelligent automata which may cooperate towards a common goal. The problem of adaptive access schemes is but an instance of the problem of community decision-making in real time. One would like to have a theory which can guide us towards efficient solutions to the general problem.

1.4.2 ANALYSIS OF RESOURCE SHARING PROCESSES

The price of intelligence is constantly decreasing while the price of other resources, in particular communication resources, is increasing. Effective resource sharing algorithms can be implemented in terms of computer-controlled decision mechanisms. The problem of analyzing the performance of interfering queueing processes is a key to the understanding of sophisticated sharing mechanisms. Classical queuing theory does not provide a sufficient set of tools to solve the problem of interference. This has been our major motivation in exploring some typical interfering service mechanisms of PRNETs.

Future research should be carried out to generalize the mathematical machinery to attack the general interference problem. The idea should be to develop a sufficient understanding of the geometry of the mathematical problem so that practical approximate solutions can be carried out with some ease.

The effort should be directed towards an "algebraization" of the interference problem, and towards a physical interpretation of the solutions.

1.4.3 THE PROBLEM OF CAPACITY

Our discussion of capacity has two objectives. First, to explore possible definitions of the capacity notion in a multi-hop environment. Second, to explore the combinatorial structure of the notion of capacity.

Future research should be carried out to explore the relation between the geometry of hearing and capacity (see [SYLV78] for many interesting results in this direction), between the routing mechanism and the capacity and to define a notion of point to point capacity in a PRNET. A deeper problem is that of establishing necessary conditions for stability of interfering processes so that a more accurate notion of capacity can be developed (rather than the heavy-traffic capacity which we use).

A problem which is completely open is to characterize the intrinsic limitations on channel usage which are imposed by the distributivity of the information and decisions required to resolve the conflict of access rights.

A possible approach is to replace the notion of centralized optimality with a family of decentralized performance criteria which approximates the global objective monotonically. For instance, start with the notion of Pareto-optimal [LUCE67] throughputs and consider the allocation of slot "value" to different PRUs, as the index of decentralization. If the family of optimal behaviors can be indexed by a proper "decentralization" parameter, then we should be able not only to solve the general problem of capacity but also, we could develop optimal decentralized allocation policies.

1.4.4 SUMMARY

The problem of decentralized adaptive control mechanisms in general, and the problem of adaptive decentralized allocation control of spatially distributed resources in particular, will most probably be among the most important problems which dominate the field of computer networks. Future research should concentrate in developing both practical solutions, as well as solid methodologies to analyze and optimize the performance of the control policies.

2. *ADAPTIVE ACCESS SCHEMES.*

2.1 THE PROBLEM OF AN ACCESS SCHEME.

2.1.1 PRNETS POSSESSED BY A DAEMON.

Let us consider a PRNET serving a population of users, whose demand for a communication-path service is random. The communication protocol is responsible for the control of the allocation of the channel resource, i.e., time#bandwidth, among the demanding packets. The allocation policy consists of the following decisions:

1. Which PRUs may transmit at each moment? We call this decision the access right.

2. Which packet in each eligible PRU gets transmitted? We call this decision the priority scheme.

3. For each transmitted packet, to which PRU is it routed? We call this decision the routing decision.

The objective is to minimize the expected delay of packets.

We use the name access scheme to designate the algorithm responsible for the first decision. In what follows we ignore the priority assignment completely and touch the routing decision only superficially. Both problems are understood in some limited sense. Moreover, in the context of PRNETs the first decision seems to be the crux of the design problem. Our main concern, therefore, is the problem of designing an access scheme.

Any access scheme is basically a scheduler of the communication channel [KLEI77]. That is, a service algorithm for communication path demands. The allocation of channel access rights to the different PRUs poses five major problems:

1. **Resource waste:**

   The transmission of one PRU may be "zapped" by the transmission of another. Therefore, if two conflicting PRUs decide simultaneously that they have access rights, the channel is wasted in a "*collision*". If, on the other hand, each busy PRU decides that he does not possesses an access right, then the channel is wasted in silence; i.e., an "*empty*" slot.

   We consider collisions and empty slots (that is, only those empty slots that could have been used) to be *allocation errors*. Both error types stem from a disparity between the allocation of access rights and the demand for those rights (as reflected by the business configuration).

2. **Bursty traffic demands:**

   The demands for a communication-path service are random and bursty. Therefore it is impossible to establish a predetermined policy for allocating access rights which meet future demands properly. That is, any predetermined decision mechanism introduces many allocation errors (i.e., collisions and empties)

because of the unpredictable state of demands.

3. **Distributed state information:**

The information about the instantaneous state of the communication demands is distributed among the PRUs. A typical network member possesses only a limited information about the needs of his comrades. Therefore, the information required for perfect decisions (i.e., perfect scheduling of the demands with no allocation errors) is not available a-priori to any decision maker.

4. **Coordination:**

A decentralized decision mechanism requires that the different PRUs coordinate their individual choices of strategy. Even if all PRUs have had perfect state information, they would still produce allocation errors if they do not coordinate their individual decisions. The situation is similar to that which arises when two people try to cross a narrow door without coordinating their movements; if both are polite (i.e., choose an "after you" policy) the door is left empty; if both are "rude" (i.e., try to push their way) they collide at the door, no one crosses.

5. **Information exchange is expensive:**

To solve the problems of distributed information and coordination

of decentralized processes, the PRUs may wish to exchange control information. An information exchange process may be expensive in terms of both delayed decisions, and consumption of the very communication resource, that we wish to utilize by the control overhead.

In short, it is required to have a demand-allocation service algorithm, but the ability of the PRNET to process such an algorithm is limited by the distributivity of both the information required for decisions and the decisions themselves.

To develop some insight into the problem, let us examine a few idealized approaches to the solution.

The first approach is to employ the services of a daemon. The daemon is aware of the instantaneous state of the queue at each PRU. It has full sovereignty over the network resources. The daemon instructs each PRU when and where to transmit. The objective of the daemon is to minimize the expected delay of packets.

Alas, even daemons have problems solving NP-hard problems such as the scheduling problem. When the network is large the computational complexity of optimal channel scheduling is practically intractable.

Being unable to solve the optimal scheduling problem, the daemon may choose a suboptimal solution. Let us assume that the daemon decides the routing independently of the channel access right. Once routing has been decided, the daemon may wish to allocate transmission rights over the channel

so that no collisions occur. Let us see what it takes for an access scheme to be an optimal collision free scheme.

The problem that our daemon faces is that of separating the busy PRUs into a minimal number of non interacting sets. Specifically, in a multi-hop network, a PRU PR_1 is said to *interfere* with another PRU PR_2, if PR_1 is heard by the immediate destination of the transmissions of PR_2. Interference is an asymmetric binary relation. Consider the graph of interference relation between the busy PRUs. A set of busy nodes which is collision free is an independent set [HARR69]. We wish the set of PRUs which are given access right to be maximal. That is, it is impossible for any other PRU to join without a collision to occur. Finally for optimality it is required that the set of PRUs which are given transmission rights is of a maximal cardinality. For if it were not maximal, then a larger number of packets could have been serviced during the same slot. Therefore we could have had a faster service mechanism.

To summarize our finding, if the daemon wishes to develop an optimal collision free access scheme (for a given routing mechanism), he has to solve another NP-complete problem [KARP72] that is, the problem of finding an independent set of maximal cardinality. The best known algorithm [TARJ6] requires $o(2^{n/3})$ steps, where n is the size of the graph. Therefore optimal collision free access schemes will usually be computationally infeasible; suboptimal solutions should be developed.

We may continue the reduction process until the daemon will face a suboptimal goal which may be solved in a reasonable time. For instance, routing may be along a minimal spanning tree, where the "length" of each edge

is the size of the queue in front this edge. Conflict resolution may be obtained by allowing any subset of non-interfering busy PRUs to transmit. Both problems may be solved in a polynomial time. Other suboptimal policies may also be considered but, the sad conclusion is that even daemons cannot process an optimal real-time allocation control policy.

2.1.2 EXORCISING THE NETWORK DAEMON.

In the previous section we saw that optimal daemons are, computationally speaking, unfeasible. Now that we are ready to settle for suboptimal solutions, the very use of a daemon is questionable.

First, daemons are not reliable. Leaving the control of the network at the discretion of a daemon may result in a demonic behavior upon failure. Second, even suboptimal daemons are not free nowadays. In fact even daemons with modest requirements of information for decision have to use a finite time to gather the information and to make their decisions known to the PRUs. This limits the ability of centralized decision making to adapt. By the time the PRUs learn about the "suboptimal" decision of the daemon, it may already be obsolete. The speed of adaptivity is limited by the speed of control information propagation. Third, the exchange of control information employs the very resource to be controled. Thus, through the coordination process the daemon may obstruct the very channel which he tries to preserve. The conclusion is that the network daemon should be exorcised. The decision making process should be distributed among network members.

Exorcising the daemon is anything but an easy task. It requires that we solve the coordination problem of the independent decisions made by network members. The ability of network members to arrive independently to an harmonious set of decisions is limited by the amount of information that each member possesses about the status and the decisions of his fellows. The relation between the time spent to coordinate the decisions and the performance of the resulting algorithm, should be the subject of an as yet undeveloped theory of

coordination complexity (see however [SPIR77, DALA77] for an initial attack on the coordination complexity of finding a minimal spanning tree - for routing - in a network). Let us describe some of the problems that we expect such a theory to resolve quantitatively.

First, distributed adaptive control algorithms usually involve two interwoven hierarchies; space and time. That is, some parts of the algorithm adapt rapidly to instantaneous local events; some parts adapt to the short range statistical behavior of the immediate environment at each decision maker; some parts adapt slowly to long range statistics of the global network state. How should we characterize the class of all hierarchical control strategies? How should we correlate the time-space hierarchy to the distributed observation process?

Second, distributed state observations imply that each PRU sees a different part of the global picture. It is possible to exchange some information between neighboring PRUs to coordinate the observations. The exchange of information enhances the observation of each PRU; however, it has a high price. How should we design low cost, effective observation exchange mechanisms? For a given amount of acceptable rate of observations updates, what is the best information to exchange? What is the best usage of the information available to each decision maker? What are the performance bounds due to the distributivity of the observation process?

Third, the common objective entails that the PRUs try to reach a maximally harmonious set of decisions. How should that be done? What are the performance bounds due to partially coordinated decisions (because of the

distributivity)? What is the optimal solution to the problem of partial coordination?

Finally distributed decision algorithms may be very sensitive to small perturbations of the observation and coordination processes. How sensitive is a given algorithm? How can we reduce sensitivity and increase the robustness of a given algorithm?

The closest models to attack problems of this nature, are those used in mathematical economics. One could easily see the similarity of our problems to those of team decision theory. However the theory of team decisions seems to be yet in its infancy. The book by R. Radner and the late J. Marschak [MARS72] is an excellent reference. Presently the theory hardly provides the tools for attacking the problems presented above. Moreover, it does not provide a characterization of the optimal decision laws (beyond some simplistic Bayesian decision schemes) and the models which it uses pose insurmountable combinatorial difficulties.

In the absence of a suitable theory, we have to apply both, intuition (+ analysis) and measurements, in order to develop suboptimal, adaptive control algorithms. In so doing, we shall try to adhere to the spirit of team decision theory.

The need for adaptive control is particularly acute in the case of PRNETs serving a bursty population of users. Indeed, a deterministic allocation of the channel such as Time Division (TDMA), or Frequency Division (FDMA); will waste most of the channel most of the time. An arriving packet has to be

satisfied with its small fragment of the channel, while most of the channel is reserved for dormant users.

On the other hand, if we try to adapt the allocation of the channel to the need of the users, imperfect distributed decisions may lead to the consumption of the resource by colliding packets. We would like to develop access schemes that adapt to both the immediate needs of users and to the total load on the system. Adaptivity should be obtained with only a minimal exchange of coordination messages. Finally, the access scheme to be developed should be decentralized and robust.

In what follows we develop a new class of adaptive access schemes based on one scheme which we call the *Urn scheme*. Our schemes follow the above guidelines and use only minimal information and coordination. It is thus possible to use them for multi-hop networks, where most known access schemes are either unimplementable or non adaptive. Moreover, we shall prove that our *Urn access scheme is the best possible among all schemes in which the PRUs may use only symmetric state information for decisions.*

Our approach is basically to reduce the information available to the network daemon, then exorcise the daemon, leaving the decision making to the PRUs.

2.1.3 COMPARISON WITH PREVIOUS WORK.

The literature on the subject of access schemes is rich [KLEI77]. Traditional channel allocation policies use a fixed, predetermined allocation policy, such as Time and/or Frequency Division. Deterministic apriori channel allocation is an excellent service policy for a steady, predictable, traffic demand. However, when it comes to servicing the traffic demands of a computer network, predetermined channel allocation is a poor way to serve such *bursty* traffic.

The servicing of a bursty communication demand requires that the allocation policy adapt to the immediate need of users. This has been achieved, to some extent, by the ALOHA policies [ABRA72, ABRA73, ROBE72]. The ALOHA allocation of the channel is obtained by a "blind channel grabbing" mechanism. That is, each PRU may decide to grab the channel and transmit a packet. To resolve conflicts between users which try to grab the channel simultaneously, a randomized retransmission policy is employed when conflicts occur. The ALOHA policies constitute an ingenious allocation policy when the traffic is very low. However, even for a low traffic, ALOHA policy is unstable [LAM74, FERG75, CARL75]. When the population of users is large the collisions will eventually build up and the channel becomes congested.

Some control mechanisms have been proposed to prevent ALOHA schemes from becoming unstable [LAM74, FERG75, FAYO76, GERL77, KLEI78]. All these schemes require extra information about the state of the channel. Even with these built in controls, the ALOHA schemes can utilize at most 1/2e (~36%) of the channel, if the the channel is unslotted, and 1/e (~18%) if the channel is slotted. It cannot handle heavy traffic.

The limitations of "blind grabbing", i.e., instability and low capacity, caused a search for better schemes. Some of the schemes that were considered are simple improvements of ALOHA policies. The idea is to permit grabbing but replace blindness with as much insight, that a PRU may get through listening, as possible. This category includes Carrier Sensing and Busy Tone multiple access schemes [TOBA75]. The success of such schemes depends upon the ability of a PRU to use the information gained by listening to the channel at his place, to infer about the state of the channel at his destination. If the hearing graph is a complete graph (i.e., all PRUs hear each other) Carrier Sensing is a good solution. However, if the hearing topology is different, sensing is not enough.

At the other end of the spectrum there is a collection of schemes which use a perfect state information to allocate the channel. This category includes reservation schemes, polling, MSAP, etc... [ROBE73, KLEI77, SCHO76]. To gain perfect state information, these schemes require a subchannel for announcements and control information. The deficiency of these schemes is the over-organization. First, too much time and channel may be consumed in order to coordinate the allocation of transmission rights. Second, the control subchannel needs to be shared too, often requiring a collision free multiple access scheme to the control subchannel. Third, the control information is usually required to be delivered perfectly, introducing a strong sensitivity to errors over the control subchannel. Finally, the amount of channel used for control is a monotonic function of the total system size (or the number of busy PRUs). Perfect information and perfect allocation require a complex centralized control mechanism. Therefore, these schemes are unsuitable for a multi- hop PRNET and may possess a low robustness even in a one hop environment.

Still another class of schemes is that of "learning schemes". The PRUs learn about the state of the system through trial-error mechanisms. This category includes the probing schemes of J. Hayes [HAYE77] and tree mechanisms of [CAPI78]. Both schemes are essentially similar. The state of the system is learned by the members through the history of collisions. The resolution of conflicts is eventually achieved by a binary search technique. Learning schemes have advantages in an environment where the information used in the learning process may be available to and trusted by the decision makers. The reliability of multistage decision processes decreases exponentially in the number of stages. Therefore it is expected that learning schemes may not be robust. However, the question of reliability is yet to be solved.

Apart from the question of reliability, it seems that learning from the history of collisions provides only a small improvement over Slotted ALOHA. For instance, the capacity of the system improves to 0.43 from 0.36 [CAPI78]. This shows that extra information is required if we are to improve ALOHA significantly.

Recently, the invention of new schemes has become a fad. Thus, the introduction of a new class of schemes requires an apology. First, most adaptive schemes are unsuitable to serve in a multi-hop PRNET, the main problem being over-organization. Second, variants of ALOHA schemes, that can be easily built into a multi-hop PRNET, require control and still do not adapt properly to a heavy or even medium traffic. The problem that we tried to solve was this: *can one develop a scheme which requires the same control information as ALOHA, retains the same simplicity and robustness, but adapts better?* Our Urn scheme seems to provide a

positive answer and can be proved optimal (among policies using symmetric state information). Third, we wanted to understand the relationship between the amount of information used for decision and the performance. How well can a PRNET perform when the allocation of the channel is subject to errors due to partial state information? As far as this deeper question is concerned, we feel that we are still far from the answer.

Our proposed Urn scheme uses the same information that some of the controled ALOHA schemes use but performs significantly better. Indeed, under light traffic the performance of the Urn scheme converges to that of ALOHA; however, when the traffic is heavy the Urn scheme converges to TDMA. When the traffic is medium the Urn scheme gives better performance then both methods. Also, our scheme permits imperfect allocation, i.e., collisions and empty slots, therefore retaining the robustness of the ALOHA schemes. Finally, the Urn scheme does not impose a limit on the useful capacity of the channel; in heavy traffic it is possible to utilize the full capacity. All of the above statements remain true even when the number of PRUs grows to infinity.

2.2 THE URN SCHEME

2.2.1 HOMOGENEOUS, SYMMETRIC INFORMATION.

In what follows we shall consider one-hop PRNETs synchronized to slots whose duration equals packet transmission time. At each time slot t the state of the service demand is completely described by the vector $Q^t \triangleq (Q_1^t, Q_2^t, \ldots\ldots, Q_N^t)$, where Q_i^t is the number of packets queued for service in the buffer of the i-th PRU PR_i at the begining of slot t. N is the total number of PRUs in the system. The decision, which PRUs get the right to transmit in the next slot, is our main concern. Let us consider the major elements of the decision making process.

2.2.1.1 State Description.

A good choice for a description of the state should contain all the relevant details which influence the queueing process, so that it may be described as a Markov chain. The process Q^t can serve this purpose if we make the assumption that both, the service and the arrival mechanism are memoryless. In particular, we preclude all decision schemes which may learn from the history of Q^t.

The state vector Q^t, contains all the information which is sufficient to generate a Markovian description of the queueing process. However, this information is more than that which the decision of access rights actually requires. The information which is relevant to the decision making process is described by the state of occupancy vector $B^t \triangleq (B_1^t, B_2^t, \ldots\ldots B_N^t)$, where $B_i^t \triangleq \mathrm{sgn}(Q_i^t)$ assumes the value 1 when PR_i is busy and 0 otherwise. Unfortunately even this information is only partially available to the decision makers; apriori each PRU knows his state only.

2.2.1.2 The Set of Available Strategies.

At each slot a busy PRU may decide to transmit or not. These are the only *pure* strategies available to him. A *randomized strategy* for the i-th PRU, PR_i, is the conditional probability P_i that he decides to transmit at slot t given that he is busy. Looked another way, P_i may be considered as the unconditional probability that PR_i has a transmission right over slot t; he transmits iff he possesses both a packet ready for transmission and a transmission right. The set $\Lambda \triangleq [0,1]^N$ is the set of all instantaneously available network strategies. A decision strategy is a map of the information used by the decision makers into the set Λ. To describe the set of strategies we should specify what we mean by "information used by the decision makers".

2.2.1.3 Objective.

We replace the objective of minimizing the expected delay with that of maximizing the throughput. This last objective greatly simplifies the argument and the computation. It is possible to show that the two objectives are equivalent under some general assumptions on the nature of the arrival and service processes. We shall not dwell upon the details of the proof. However, the idea is quite simple. Indeed, if the service mechanism (the access scheme) was lossless then the overall expected delay would have been invariant to changes in the service priorities (which would have been the only free parameter of choice). Such a result is a simple expression of conservation laws [KLEI76]. When the service may be lost (say, in the form of "empty" and "collision" slots) we need a generalization of the conservation laws in the following form: a priority scheme minimizes the overall expected delay iff it maximizes the expected rate of service, i.e., the throughput. The derivation of

conditions under which this generalization is correct, is beyond the scope of this dissertation and left for future research.

(A proof may be obtained if we consider the structure of the evolution equation of the queueing process Q^t:

$$Q^{t+1}=Q^t+C^t-S^t$$

where C^t is a vector whose i-th coordinate describes the number of packets arriving to PR_i at slot t; S^t is a binary vector whose i-th coordinate is 1 if PR_i delivered a packet successfully at slot t (i.e., the instantaneous throughput). The access scheme can not affect the value of C^t. The only control that we exercise over the network is through S^t. The objective of minimizing the expected delay is identical to that of minimizing the expected number of queued packets, by Litttle's result [KLEI75]. The later objective can be expressed in terms of a minimization of the expected value of $|S^t| \triangleq \sum |S_i^t|$, by arguments identical to those of J. Marschak and R. Radner [MARS72] (chapter7).

2.2.1.4 **Information Used by Distributed Decision Algorithms.**

In deciding the best strategy at slot t, the decision makers can only use the state information *available* to them. However, a PRU may decide not to use all the information available for him. This is the case, for instance, when the use of some details will not improve the performance significantly while increasing the computational effort by far. Therefore we choose as our point of departure to consider the information that an individual PRU *actually uses* for decision at any given time slot.

The information which is used by each individual may be classified under

two categories:

a. *State information:*

Each member of the community observes some function of the state of the network. Network members may acquire further information through a limited exchange of messages.

b. *Coordination information:*

In order to reach harmonious decisions, network members should have some information about the decisions taken by their fellow members. Such information may be made available through some limited information exchange between members, a predetermined set of rules, or a combination of both.

It is possible to lump the decisions and the state vectors into one large "augmented state". We choose to separate the two momentarily, only to demonstrate the importance of the homogeneity assumption to eliminates the needs for coördination.

The state information which PR_i uses for his decision at slot t, may be described by his subjective probability measure over the states of occupancy. We call this probability measure, π_i: the *information measure* of PR_i. In what follows, we shall describe our assumptions about the information measures π_i.

Our first assumption is that of *homogeneity*. That is, the information which each PRU uses for decision at each slot, is the same for all PRUs. Formally

$$\forall b \in B, \quad \pi_1(b)=\pi_2(b)=\ldots\ldots=\pi_N(b)$$

where $B \triangleq \{0,1\}^N$ is the set of occupancy state vectors.

The assumption of homogeneity reduces the need for dynamic coordination of the distributed decision making. In fact, the decision makers can be preprogramed to respond optimally to each possible information measure. Harmony is obtained because each member knows what his comrades will choose and the role that he should play. Coordination is achieved by a set of deterministic rules which are known to all network members. Note that from a practical point of view the assumption of homogoneity is, usually, only practical in a one-hop PRNET. Accordingly, we shall produce schemes to decompose a multi-hop network into a collection of one-hop systems, in each of which the decision of access-rights is homogeneized (see section 2.3).

As an example of a homogeneous information measure, consider the case of *perfect* information. Here the information measure is a delta distribution concentrated on the current state of occupancy. An optimal decision rule is *any* priority scheme which assigns a full transmission right to a single busy PRU. We may use a fixed priority assignment, a round robin or any other arbitration. The main point is that the priority rules are decided apriori; by assumption each PRU is aware of those rules. Thus, if each PRU has perfect information he can immediately find out the allocation of transmission rights. There is no need for extra communication between PRUs to coordinate the decision dynamically.

Another case of homogeneous information measure is that of no information whatsoever. That is π is the uniform distribution over B. In this case a PRU has no knowledge about his friends. An optimal stationary strategy must be stable. A simple stability argument shows that for optimality, each PRU must choose to transmit with a probability equal to 1/N (N=total system size). Indeed, if a PRU has no information whatsoever about the behaviour of his fellow PRUs (except for their total number), then, in choosing his transmission probability it must account for the worst occupancy vector that his fellow PRUs may choose. This is a typical case of max-min decision policy in the absence of prior state information [FERG67]. That is, each PRU should choose his transmission probability under the assumption that the state occupancy vector is $\underline{1} \triangleq (1,1,\ldots\ldots 1)$. Under this assumption the optimal choice of transmission probabilities to maximize throughput is $P_i = 1/N$. This policy is precisely uncontrolled Slotted ALOHA. An interesting feature of this policy is that decision process can be described as a game of the access algorithm against the communication demand. The Aloha strategy is a max-min strategy for the access algorithm.

Another case of an interesting homogeneous information is that of having a complete description of the stationary behavior of the system. That is, each PRU is made aware of a Markovian description of the state of all other PRUs. In this case each PRU may compute the equilibrium distribution for each choice of transmission policy, then choose that transmission policy which maximizes the stationary performance. The information measure of each PRU is the equilibrium distribution obtained by choosing an optimal transmission policy. It is possible to design a quasi-static algorithm (in the sense of [GALL77]) which

solves for the optimal stationary transmission policy through an iterative process. We shall not pursue this subject here, for we wish to develop an algorithm which adapts to the dynamics of channel demands, and not to the equilibrium state.

What will happen if the information measures are not homogeneous? Evidently the complexity of the decision making increases significantly. It is not enough for a PRU to estimate the actual state of the system. It is also necessary for each PRU to estimate the strategies of his fellows. This is still insufficient because other network members will base their decisions upon their estimates of his strategy. Therefore, he should estimate their estimates. Clearly such a process of estimation is computationally unrealistic. Usually a solution will be found where the network members exchange some information for coordination and ignore those parts which make their individual information measures non homogeneous. In short the problem is very hard and for practical solutions it is usually avoided. How should distributed decisions be arrived at, when the information is inhomogeneous? The answer to this problem is open and should be answered within a future theory of decentralized algorithms, to be developed.

To summarize, the assumption of homogeneous information eliminates the problem of coordination. If the information is perfect, the problem reduces to that of ordinary centralized optimization. If no state information is used, the problem becomes that of choosing a max-min strategy. At one extreme there is a full cooperation at the other extreme full distrust. The problem of characterizing the spectrum of information measures between the two

extremes, is left for a future research. We proceed to examine an important intermediate case.

Let us assume that the information is *symmetric*. That is, a PRU does not use for decision any information about the individual identity of those that are busy. Formally speaking, the common information measure π, is invariant w.r.t. the group of coordinate permutations of vectors in B. That is:

$$\pi(b_{i_1}, b_{i_2}, \ldots\ldots, b_{i_N}) = \pi(b_1, b_2, \ldots\ldots, b_N)$$

For all $b \in B$ and for all permutations $(i_1, i_2, \ldots i_N)$ of $(1,2,\ldots N)$.

We may identify the distribution π with a distribution over the equivalence classes of state vectors, w.r.t. equality modulo permutations. Each such equivalence class is completely described by the number of busy PRUs (i.e., the number of 1's in the state vector). Thus, a symmetric information is a probability measure on the set $\{0,1,2\ldots,n,\ldots N\}$, where n stands for: n busy PRUs in the network.

Finally, the problem that we face is to maximize the expected throughput, over the class of all decision functions $d:[0,\infty)\times\Omega \dashrightarrow \Lambda$ where Ω is the set of probability measures over $\{0,1\ldots\ldots N\}$ and $\Lambda=[0,1]^N$ is the set of all available strategies. *We shall restrict ourselves to stationary decision functions only (i.e.,* d *is time independent explicitly, tough it may depend upon time implicitly through the state).*

The best symmetric information that the PRUs may have, is the actual number of busy PRUs. The subject of the following sections is to develop an optimal stationary decision scheme which uses this information only. We

derive an optimal access scheme under symmetric information, then we examine the problem of implementation of the basic scheme and its variants.

2.2.2 AN OPTIMAL ACCESS SCHEME.

Let $\sim[0,1]^N$ designate the set of all randomized strategies available to the network. Here a generic strategy $\underline{q}$ is a vector whose i-th component q_i, is the probability that PR_i will *not* transmit a packet given that he is busy*. We define a few functions of the strategy which are of importance.

First let us introduce some notational conventions. An *occupancy configuration* of n PRUs is a subset of n indices taken from {1,2....N}. A given occupancy configuration may be uniquely identified with a vector of indices $\underline{i}=(i_1,i_2,\ldots\ldots,i_n)$ where $i_1<i_2<\ldots\ldots<i_n$. Let I^n describe the set of all such index vectors. Let I_i^n describe the set of all occupancy configurations, which do not include the index i. Equipped with these conventions we define the following functions:

$$(2.2\text{-}1)\qquad E^n \triangleq \begin{cases} \sum_{\underline{i}\in I^n} \prod_{j=1}^{n} q_j & n>0 \\ 1 & n=0 \end{cases}$$

E^n is, up to a normalizing factor, the expected fraction of empty slots (or probability of an empty slot) conditioned on the number of busy PRUs being n.

$$(2.2\text{-}2)\qquad E_i^n \triangleq \sum_{\underline{i}\in I_i^n} \prod_{j=1}^{n} q_j$$

E_i^n is, up to a normalizing factor, the expected fraction of slots during which PR_i may expect no interference, conditioned on the number of possible interfering PRUs being n.

---

*Choosing the probabilities of silence as the decision parameters, rather than the transmission probabilities, is merely to simplify formulae

In terms of these functions the expected fraction of slots during which a packet is delivered successfully, conditioned on the number of busy PRUs being n, is given by:

$$(2.2\text{-}3) \qquad S^n = \left[1/\binom{N}{n}\right] \sum_{i=1}^{N} (1-q_i) E_i^{n-1}$$

We call S^n the *conditional throughput*. It is easy to prove that an optimal strategy, i.e., a strategy which minimizes the delay, must maximize the conditional throughput. This follows from the results presented in chapter 7.1 of [MARS72].*

Therefore let us look for the strategy $\underline{q}$ which maximizes the conditional throughput. First let us derive som_e simpler expressions for S^n, which separate the symmetric terms from the non symmetric.

A simple application of the principle of inclusion- exclusion [RIOR58] yields the following identity:

$$(2.2\text{-}4) \qquad E_i^n = E_i^{n-1} - q_i E_i^{n-2} + q_i^2 E_i^{n-3} - q_i^3 E_i^n + \dots + (-q_i)^{n-1} E^0$$

Let us use this expression for E_i^{n-1} in 2.2-3. We get the following expression for the conditional throughput:

$$(2.2\text{-}5) \qquad S^n = \left[1/\binom{N}{n}\right] \sum_{i=1}^{N} (1-q_i) \sum_{j=0}^{n-1} (-q_i)^j E^{n-j-1} =$$

$$= \left[1/\binom{N}{n}\right] \sum_{j=0}^{n-1} (-1)^j E^{n-j-1} \sum_{i=1}^{N} (1-q_i) q_i^j$$

To gain some geometrical insight into the structure of the function S^n let us

*See also the comment to that effect in section 2.2.1 under the description of the objective.

examine some simple examples of S^n. The function S^1 is given by the expression:

$$\text{(2.2-6)} \quad S^1 = (1/N)\sum_{i=1}^{N}(1-q_i) = (1/N)\left[N - \sum_{i=1}^{N} q_i \right]$$

This is a linear function of $\underline{q}$ so the optimum strategy must be an extreme point of Λ. Clearly $\underline{q}=(0,0,....0)$ is the best strategy, yielding a conditional throughput $S^1=1$.

When n=2, the conditional throughput becomes:

$$\text{(2.2-7)} \quad S^2 = [2 / N(N-1)]\left[\left(\sum_{i=1}^{N} q_i\right)\left(N - 1 - \sum_{i=1}^{N} q_i\right) + \sum_{i=1}^{N} (q_i)^2 \right]$$

Consider the case of two PRUs only, i.e., let N=2. The surface $S^2(\underline{q})=q_1(1-q_2)+q_2(1-q_1)$ is depicted in Figure 2.2-1. The point $\underline{q}^0 \triangleq (1/2,1/2)$ is a stationary point of S^2. As a matter of fact, if we consider the line of symmetric strategies, $q_1=q_2$, then $\underline{q}^0$ is the best *symmetric* strategy (for symmetric strategies, $S^2(\underline{q})=2q(1-q)$, having a maximum at $q=1/2$). However, $\underline{q}^0$ *is not the best available strategy*. Indeed, $\underline{q}^0$ is a saddle point of S^2. The best strategies are $\underline{q}^1=(1,0)$ and $\underline{q}^2=(0,1)$, that is, one PRU should get a full right of transmission while his fellow gets no right whatsoever. If both PRUs are aware of the fact that both are busy, then they can decide who should transmit through a preprogrammed priority mechanism. There is no need for a randomized decision.

Now let N= 3 and n=2. The cube of strategies is depicted in Figure 2.2-2. $S^2(\underline{q})$ is a symmetric function of $\underline{q}$. Let us search first for optimal points of S^2 inside the cube. Possible candidates must lie on the main diagonal. Thus we

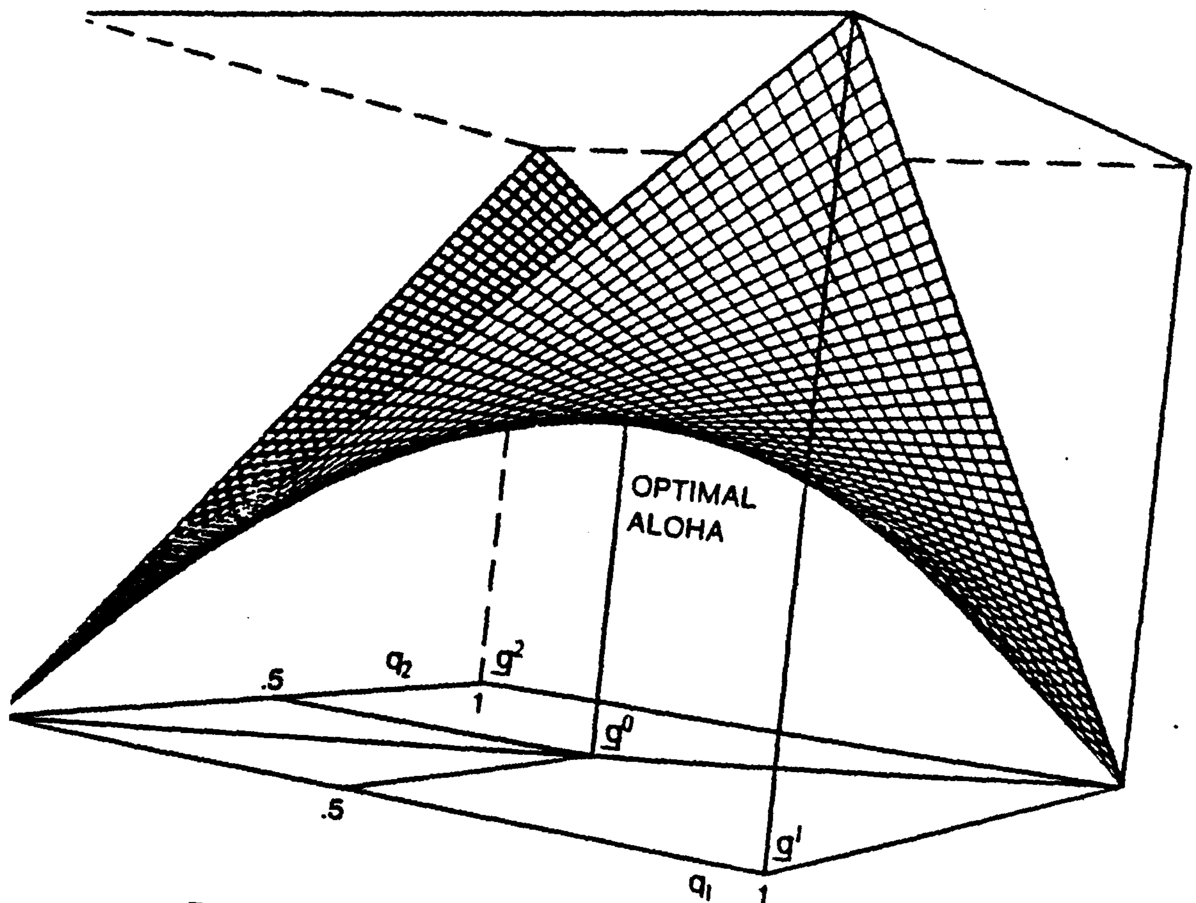

Figure 2.2-1: Throughput Surface for Two Busy PRUs

seek to maximize the restriction of S^2 to the main diagonal, i.e., $S^2(\underline{q})=3(1-q)q^2$. It is easy to see that the point $\underline{q}^0=(2/3,2/3,2/3)$ obtains an optimal throughput among all points on the main diagonal (symmetric policies). This point corresponds to optimal ALOHA policies; it obtains a throughput $S_0=3\times(1/3)\times(2/3)^2=4/9$.

Let us try to improve the performance considering policies on the boundary. Fix q_1 to be 0. On the resulting face of Λ the function S^2 is symmetric in the other two policies. A stationary point must lie on the diagonal $q_2=q_3$. Therefore we seek to maximize the restriction of S^2 to this diagonal, $S^2=(2/3)q(2-q)$. Clearly the optimal choice of q is q=1. The corresponding point $\underline{q}^1=(0,1,1)$ obtains a higher throughput than $\underline{q}^0$, i.e., $S_1=1-P[\text{Both } PR_1 \text{ and } PR_2 \text{ are busy}]=1-(1/3)=2/3$ while $S_0=4/9$. Now consider S^2 restricted to the face (edge) $q_1=q_1=0$. On this face $S^2(\underline{q})=(2/3)q$ is a linear function obtaining its maximum value at the corner $\underline{q}^0=(0,0,1)$, the maximum being $S_2=2/3$ again. We may continue the search over the other faces. For instance when $q_1=1$ the optimal point is the corner (1,0,0) and so on. We conclude that in the case N=3 n=2, optimal policies are *pure* policies, that is any of the following corners of the cube: (0,0,1), (0,1,0), (1,0,0), (0,1,1), (1,0,1) and (1,1,0). An optimal policy consists of letting one member (two members) have a full transmission right and the other two (one) none.

To summarize our findings, the symmetric strategies do not necessarily provide the maximal throughput. The maximum throughput may be obtained by an *asymmetric* policy, i.e one which assigns full transmission rights to a subset of the PRUs and no right to all the others. We shall now generalize these

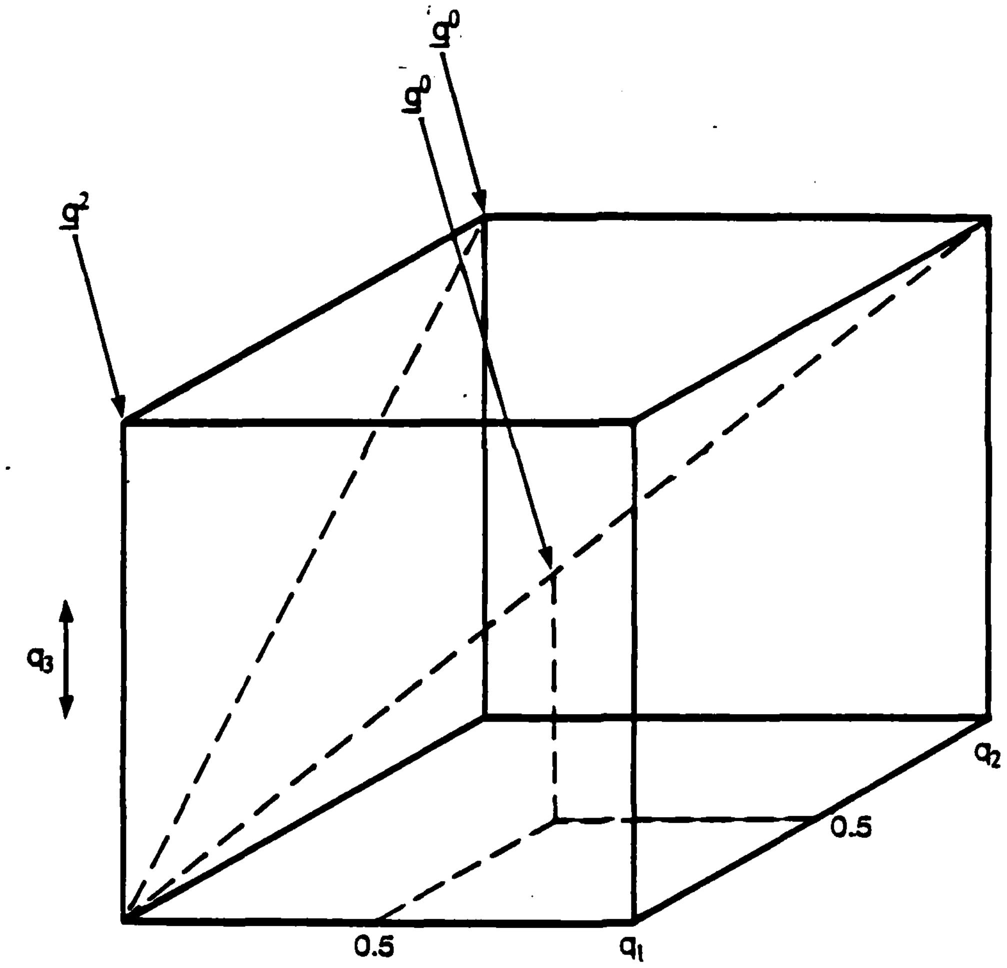

Figure 2.2-2: The Cube of Strategies for N=3 PRUs

observations to S^n.

We wish to find a strategy $\underline{q}$ which maximizes the throughput function S^n. Unfortunately the throughput S^n is not a "nice" function; that is, it is not concave or convex in $\underline{q}$. Later we shall show how to reduce the problem to a convex maximization. In the meantime we first search the optimal strategy in the *relative interior* of all the faces of the hypercube of strategies Λ. The search will contribute to our understanding of the solution.

The complexity of the search may be reduced significantly if we utilize the symmetry of S^n. Indeed, if we restrict some (possibly none) individual strategies q_i to be pure, i.e., 0 or 1, S^n becomes a symmetric function of the remaining coordinates. A stationary point of the restricted S^n is necessarily symmetric in all the remaining coordinates. Therefore we only have to examine strategies which are partially pure and partially symmetric. Geometrically speaking, we should only examine the local maxima of the restriction of S^n to the main diagonals of the faces of the cube Λ.

In what follows we shall consider strategies which are partially symmetric and partially asymmetric. We first optimize the strategy of the symmetric users. Then we search for the optimal number of asymmetric users. The search will finally lead us to a pure strategy (i.e., a strategy where some PRUs need to keep silent while others are given full access rights). The optimal strategy is an extreme (corner) point of the hypercube of strategies Λ, i.e., a pure strategy.

Let us consider symmetric strategies on a face of Λ determined by a choice of k PRUs to transmit, say $\underline{q}=(0,0,0...0,q,q,...q)$. Let m be the number of symmetric

PRUs, k+m=N. We shall use the notation $S_m^n(g)$ to denote the conditional throughput obtained by the restricted strategy. We shall first compute the function S_m^n, then maximize it.

First note that

$$(2.2\text{-}8)\quad \sum_{i=1}^{N} (1-q_i)(q_i)^j = \begin{cases} k+m(1-q) & \text{if } j=0 \\ m(1-q)q & \text{if } j\neq 0 \end{cases}$$

Also

$$E^0 = 1$$

$$E^1 = mq$$

$$E^2 = \binom{m}{2} q^2$$

$$\vdots$$

(2.2-9)

$$\vdots$$

$$E^r = \binom{m}{r} q^r \quad \text{for } r\leq m$$

Thus, the restricted throughput may be computed from equation 2.2-5 to give:

$$(2.2\text{-}10)\quad S_m^n(g) = E^{n-1}[k+m(1-q)] + \sum_{j=1}^{n-1} (-1)^j E^{n-j-1} m(1-q)^j =$$

$$= \binom{m}{n-1} q^{n-1}(N-m) + m(1-q)q^{n-1} \sum_{j=0}^{n-1} (-1)^j \binom{m}{n-j-1}$$

Using the identity:

$$\text{(2.2-11)} \qquad \sum_{j=0}^{n-1} (-1)^j \binom{m}{n-j+1} = \binom{m-1}{n-1}$$

we get:

$$\text{(2.2-12)} \quad S_m^n(q) = \frac{\binom{m}{n-1}}{\binom{N}{n}} q^{n-1} \left[(N-n+1) - q(m-n+1) \right]$$

For instance, when k=0 (m=N) equation 2.2-12 reduces to the well known form

$$\text{(2.2-13)} \quad S_N^n(q) = \frac{\binom{N}{n-1}}{\binom{N}{n}} q^{n-1}(N-n+1)(1-q) = nq^{n-1}(1-q)$$

Differentiation of the function $S_m^n(q)$ w.r.t. q yields the following value of the unique stationary point q^0:

$$\text{(2.2-14)} \quad q^0 \doteq \left(\frac{n-1}{n}\right) \left(\frac{N-n+1}{m-n+1}\right)$$

The respective throughput is given by:

$$\text{(2.2-15)} \quad S_m^n(q^0) = \frac{\binom{m}{n-1}}{\binom{N}{n-1}} \left(1 - \frac{1}{n}\right)^{n-1} \left(N-n+1 \,/\, m-n+1\right)^{n-1}$$

For a later reference we shall also need the following form:

$$\text{(2.2-16)} \quad S_m^n(q^0) = \frac{\binom{n}{1}\binom{N-n}{k-1}}{\binom{N}{k}} \left(\frac{N-n+1}{kn}\right) q_0^{n-1}$$

For example, when k=0 (m=N) the optimal symmetric strategy q^0 is 1-1/n, i.e., everybody gets the right to transmit with probability 1/n . The optimal value of the throughput obtained by symmetric strategies is $(1-1/n)^{n-1}$. Both values conform to well known formulae for optimal Slotted ALOHA [ABRA73].

Let us examine the value of the optimum throughput as k increases from 0 towards N-n, i.e., m decreases from N towards n. For this purpose we examine the ratio

$$(2.2\text{-}17) \quad f_m^n \triangleq \frac{S_m^n(q^0)}{S_{m-1}^n(q^0)} = \left(\frac{m}{m-n+1}\right)\left(\frac{m-n}{m-n-1}\right)^{n-1}$$

We wish to prove that $S_m^n(q^0)$ is a monotone, nonincreasing function of m. It is enough to show that the ratio in equation 2.2-17 is smaller than 1. To show this we note that

$$f_m^1 = 1$$

$$f_m^2 = m(m-2)/(m-1)^2 < 1$$

Also, the derivative ∂f_m^n may be easily shown to be negative. Therefore, the ratio 2.2-17 is a decreasing function of n and becomes smaller than 1 for $n>1$. That is, $f_m^{n+1} < f_m^n$. Thus the stationary throughputs $S_m^n(q^0)$ keep on increasing as m decreases (k increases).

Since the decrease in the number m, of users which are given symmetric rights over the channel, increases the throughput, the question arises: how small can m become ?

The answer is obtained once we consider the expression 2.2-14 for the optimal value of q. Indeed since q^0 can not exceed 1, the following inequality follows:

$$(2.2\text{-}18) \qquad m \geq (n-1)(N+1)/n$$

from which we get:

$$k \triangleq N-m \le (N-n+1)/n < \lfloor N/n \rfloor + 1^{*} \qquad (2.2\text{-}19)$$

Therefore the optimal choice of k is the maximal integer which is not greater than (N-n+1)n. Let us, for the time being, ignore the requirement that k be an integer and assume that equality holds in the leftmost inequality of 2.2-19 (i.e., $k_{opt} = (N-n+1)/n$) then

1. $q^0=1$. That is, the symmetric users are all silent. Therefore *the optimal strategy is an extreme point of the hypercube* Λ.

2. The optimal throughput, as given by 2.2-16, assumes the form:

$$\frac{\binom{n}{1}\binom{N-n}{k-1}}{\binom{N}{k}}$$

   which is a term of the Hypergeometric distribution, describing the probability of getting one black ball from an urn containing n black balls and N-n white balls, once we draw k balls. This, seemingly unrelated, interpretation will soon be shown to provide a good model for our problem.

To conclude the discussion, the optimal strategy among all strategies in which some PRUs receive full transmission rights and the remaining PRUs get a symmetric right, is to give full transmission rights to $k_{opt} \triangleq (N-n+1)/n^{*}$ PRUs and no right whatsoever to the remaining PRUs.

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*The notation $\lfloor x \rfloor$ stands for the greatest integer which is not greater than x.

*This number is usually non-integer and therefore the result need to be further elaborated. This problem will be addressed soon.

In order to be able to claim that the above rule is the best network strategy, we should repeat the tedious process of computing the conditional throughput for strategies of type (1,1,1...,1,q,q,...,q). We have to find the best value of q. Then, we have to find the best number of silent PRUs. The process is a simple repetition of the computation above and the results are the same (i.e., the optimal value of q is 0 and the optimal number of users for which q=0 should be k=(N-n+1)/n). A shorter proof will be presented soon.

The asymmetric (pure) strategy lends itself to an easy interpretation in terms of an urn model. The PRNET may be thought of as a collection of intelligent balls, which are colored black (for busy) or white (for idle). Each ball may choose to jump out of the urn or stay inside with some individual probability. "Jumping out" of the urn corresponds to acquisition of a transmission right. The objective is to have one and only one black ball among those who choose to jump out. If the only information available is the number of black balls then the best strategy is to assign a probability 1 of jumping (i.e., q=0) for some balls and probability 0 to the rest.

Let N be the total number of balls, n is the number of black balls, k the number of balls to be drawn. We would like to choose an optimal value for k. The probability of drawing exactly one black ball is given by the value of the Hypergeometric distribution:

$$(2.2\text{-}20)\quad H(1,k,n,N) \triangleq \frac{\binom{n}{1}\binom{N-n}{k-1}}{\binom{N}{k}}$$

To find the optimal value of k, let us consider the following ratio:

$$(2.2\text{-}21)\quad g_k \triangleq H(1,k,n,N) \,/\, H(1,k+1,n,N) \;=\; k(n-k) \,/\, (k+1)(N-n-k+1)$$

It is easy to check that $g_k < 1$ as long as $k < (N-n+1)/n$ and $g_k > 1$ when the inequality is reversed. Therefore, $k = (N-n+1)/n$ would have been the optimal choice of k had we permitted (possibly) non-integral values of k. This result is identical to our previous formula derived through the search method.

We now turn to the problem of non-integrality of $k=(N-n+1)/n$. Since k can only assume integral values, the inequality 2.2-19 implies $\lfloor (N-n+1)/n \rfloor$. Also, the remaining $m=N-k$ users should assume access rights with probability $1-q^0 \sim 1/n(N-n-k+1)$ (here q^0 is given by the expression of 2.2-14). This last probability is very close to zero; it can be considered as a form of compensation for the integrality of k. That is, since we are constrained to select an integral k the actual optimal policy is a perturbation of a pure policy where PRUs that should have been silent are given a transmission right with a probability very close to 0. We shall, for practical purposes, ignore the compensation terms and use a pure policy. Since $\lfloor N/n \rfloor - 1 + (1/n) \leq (N-n+1)/n \leq \lfloor N/n \rfloor + (1/n)$, a natural choice for an approximately optimal k is $k = \lfloor N/n \rfloor$ or $k = \lfloor N/n \rfloor - 1$. Another possibility is to randomize between these two values of k. Henceforth we shall adopt the value $k = \lfloor N/n \rfloor$. We call a pure policy which allocates transmission rights to $k = \lfloor N/n \rfloor$ PRUs and no rights to the remaining PRUs an *Urn scheme*.

The probability of success (i.e., conditional throughput) for the Urn scheme is

$$S_{opt}^{n} = \frac{\binom{n}{1}\binom{N-n}{\lfloor N/n \rfloor - 1}}{\binom{N}{\lfloor N/n \rfloor}} \tag{2.2-22}$$

Figure 2.2-3 depicts the dependence of the optimum conditional throughput,

upon the number of busy PRU for a system of N=10 PRUs. The dashed curve describes the respective throughput of optimally controlled Slotted ALOHA i.e., symmetric policy. The diagonal line describes the respective performance of TDMA. The relations between the curves are not sensitive to system size and look the same for N=100.

When the network is lightly loaded, the optimal Urn policy performs comparably to optimally controlled ALOHA. When the network is heavily loaded the optimal strategy converges to TDMA (whose capacity approaches 1 as the load increases). In the range of medium traffic, it performs better than both schemes. Finally the mechanism that the Urn scheme employs does not impose limitations upon the useful capacity of the channel. True, collisions and wasted slots are still permitted (this is the price of imperfect information, i.e., homogeneous, symmetric and memoryless), however as the load increases, so does the value of our partial information; therefore the waste of the channel will decrease.

The urn model provides a simpler proof that the optimal strategy is pure. Indeed, let us condition the throughput S^n on the number k of balls which choose to jump out. S^n becomes a convex combination of Hypergeometric terms $H(1,k,n,N)$; that is, $S^n = \sum P[k \text{ balls jumped}] \times H(1,k,n,N)$ where the summation is on $0 \le k \le N$. If we maximize this convex combination over the N+1 simplex of all possible distributions of k, we have a standard convex optimization problem. The maximal throughput is obtained at those extreme points of this simplex which select $k = \lfloor N/n \rfloor$ (or $\lfloor N/n \rfloor - 1$) balls.

The optimal choice of k has another surprising property. Let us compute the

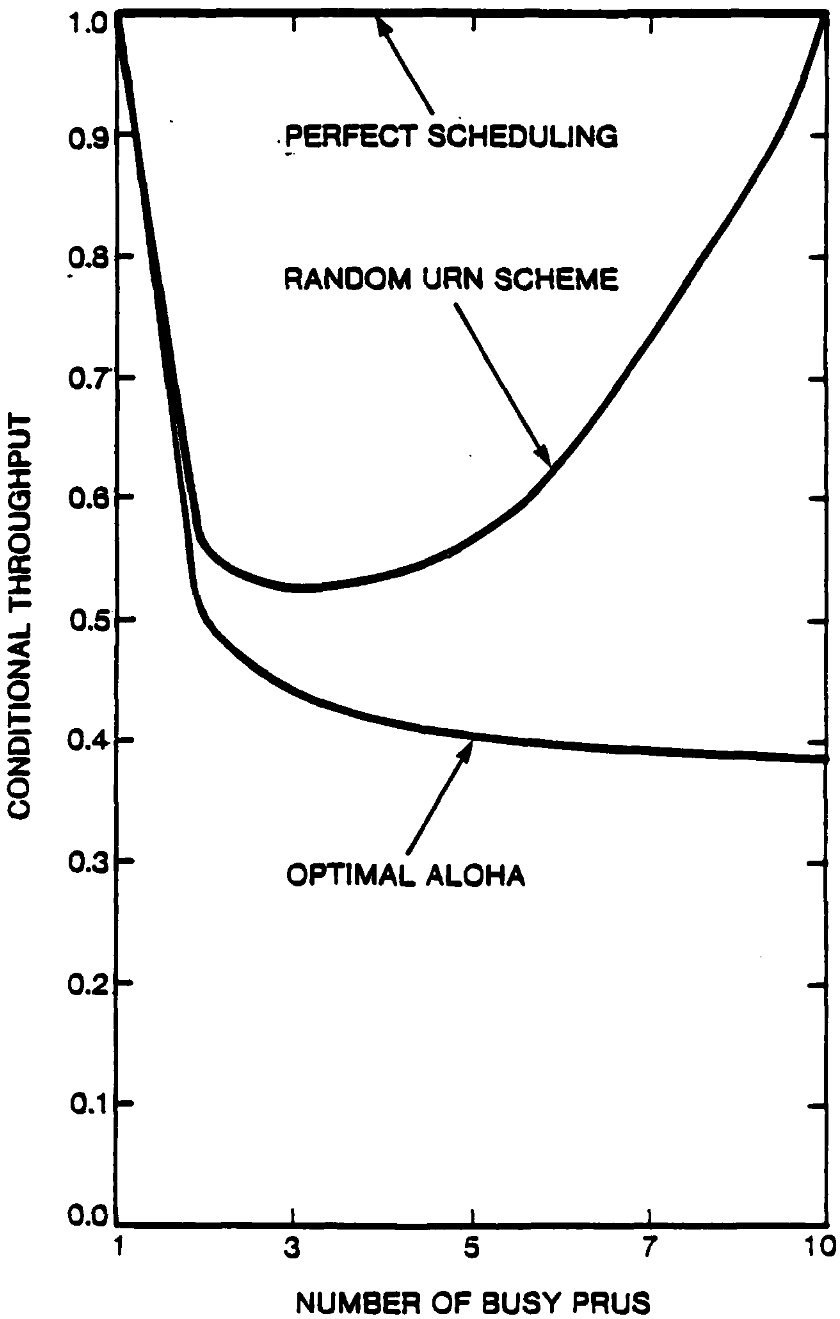

Figure 2.2-3: Conditional throughput vs. number of busy PRUs

expected number of black balls in a sample of k balls from an urn having a total of N balls n of which are black. We shall denote this expected number G(k,n,N) (G denotes the expected channel traffic, similarly to [ABRA73]). The probability that j out of the k balls sampled, happen to be black is given by the Hypergeometric distribution H(j,k,n,N). Therefore

$$(2.2\text{-}23) \qquad G(k,n,N)=\sum_{j=0}^{k} j \times H(j,k,n,N)$$

Let us use the following combinatorial identity ([RIOR68], equation 11, section 1.4):

$$(2.2\text{-}24) \qquad \binom{m}{p}\binom{r}{q} = \sum_{j=0}^{k} \binom{m-r+q}{p-j+q}\binom{j}{q}\binom{r}{j}$$

Using this last identity with $m \triangleq N-1$, $p \triangleq n-1$, $q \triangleq 1$ and $r \triangleq k$, we derive

$$(2.2\text{-}25) \qquad \binom{N-1}{n-1} k = \sum_{j=0}^{k} j\binom{N-k}{n-j}\binom{k}{j}$$

This identity can be devided by $\binom{N}{n}$ to give

$$(2.2\text{-}26) \qquad \sum_{j=0}^{k} j \times H(j,k,n,N) = (k \times n)/N$$

For the optimal choice of k this last expression is approximately 1, a result which is similar to Abramsons G=1 optimality condition [ABRA73] (also [KLEI77]).

Now let us examine the asymptotic behavior of the Urn scheme when the size of the users population N grows to infinity. Let us assume that the ratio $z[d] \triangleq n/N$ remains fixed, i.e., the probability of being busy remains fixed. As N grows to infinity, δ kept constant, the Hypergeometric distribution 2.2-20 may be approximated by a binomial distribution [FELL63].

$$(2.2\text{-}27) \qquad H(1,k,n,N) \sim k\delta(1-\delta)^{k-1}$$

The optimal Urn policy is to draw $k \sim 1/\delta$ balls. The throughput of this policy is approximated by:

$$(2.2\text{-}28) \qquad S_{opt}^{\infty} \sim (1-\delta)^{(1-\delta)/\delta}$$

As N grows to infinity the throughput of the Urn scheme is asymptotic to the expression 2.2-28. *For small δ the throughput becomes 1/e, similarly to controlled ALOHA. When the load δ approaches 1, the throughput also approaches 1. Therefore for an infinite population the Urn scheme performs as ALOHA for a small load; however it is not limited in capacity and can provide the full capacity when the load is heavy.* Figure 2.2-4 depicts the behavior of the throughput S as a function of the load δ, for the infinite-population model of the Urn scheme.

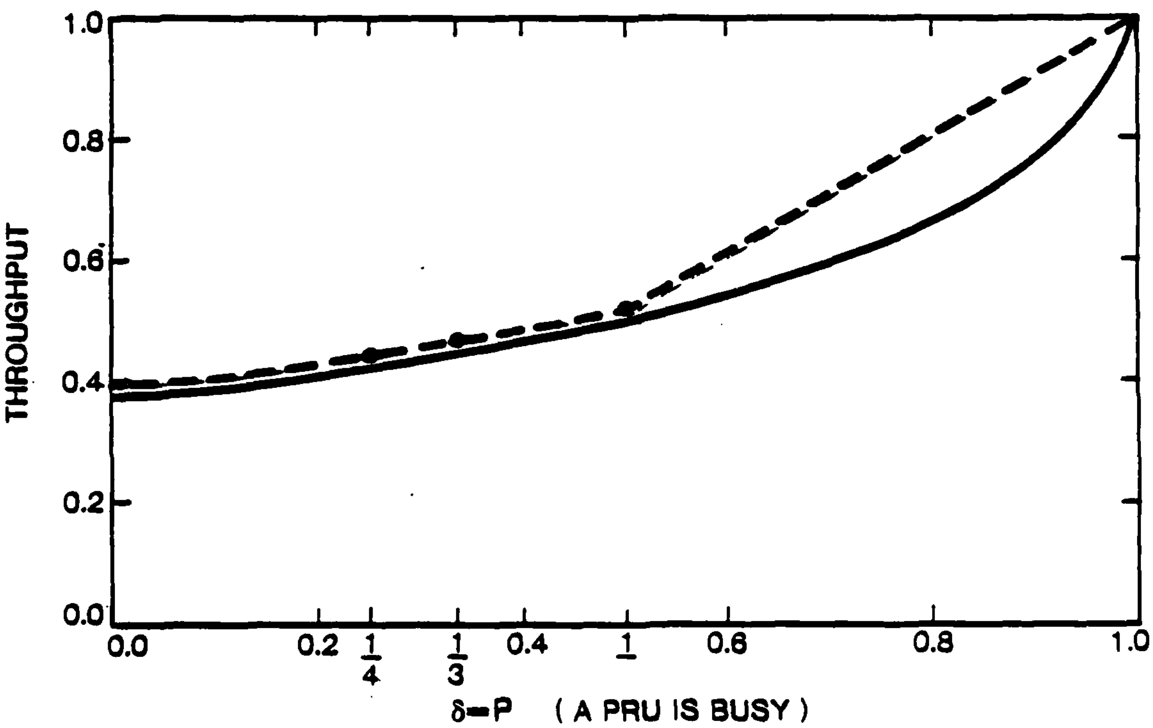

Figure 2.2-4: Throughput vs. load for an infinite population Urn scheme

In what follows, we examine the problem of implementing the optimal strategy. We analyze and compare the performance of the Urn scheme and its variants with that of other schemes.

2.2.3 IMPLEMENTATION

To implement the Urn scheme, two fundamental problems must be resolved:

1. *The problem of state information acquisition.*

2. *The problem of coordinating the distributed decision.*

When we leave the domain of theoretical models to the domain of practical algorithms it is required that we develop practical approximations to the ideal structure of the model. This is the major objective of the following section.

2.2.3.1 Acquisition of Information

Information about the state of the PRNET may be obtained by monitoring the state of the channel, and/or incorporating exchange of information between PRUs.

The information which may be acquired through channel monitoring is subject to the specific hearing topology. If the hearing graph is completely connected, i.e., each network member hears all his comrades, then channel monitoring may provide sufficient control information. However, if the assumption that all PRUs are within hearing range is false, then monitoring the channel may be insufficient.

Channel monitoring is the only way to acquire free state information. To gain any further information one has to introduce a mechanism for exchange of status and control information. Any such exchange scheme must use a *control subchannel*. It is possible to implement the control subchannel in the form of mini time slots of the channel, or a mini slice of the total bandwidth (or some combination of both). Whichever way we choose, a price must be paid for

control in terms of a portion of the communication resource lost for overhead. Moreover the control channel is, again, a multi-access broadcast channel which is to be shared by the PRUs. We face a recursive problem of designing an access scheme to the control subchannel.

In addition to the above problems, the control channel poses another problem of major importance, the problem of reliability. Many access algorithms are very sensitive to errors in the information used for decision. If the information is not available in time and /or contains errors, the algorithm may not function at all. This problem has hardly been considered for lack of quantitative theory of reliability of algorithms.

How much overhead does the exchange of information require? We have no clean way of answering this question. That is, the amount of overhead is a function of too many parameters. Therefore we use simple crude estimates. If the control subchannel is to be error free (i.e., have a negligible rate of errors) then it requires not only forward error correction (coding and decoding) overhead, but also some elimination of collisions.

If the control subchannel is to be collision free, then the size of the channel grows asymptotically like N, when perfect information is required (N is the total size of the system); or like logN when the information required is symmetric, i.e., total number of busy PRUs. (the author is, however, unaware of a scheme to achieve the later number). Moreover, the different PRUs are required to be synchronized to identify their mini share of the mini control channel. The synchronization overhead plus the coding decoding overhead plus the size of N, may result in a maxi control channel and a mini data channel.

The conclusions of the above discussion are two:

- Access schemes should be insensitive to errors in the information required by the decision algorithm. The information exchange mechanism may be trusted to a very limited extent.

- Exchange of control information should be restricted to a small subchannel.

The design of adaptive access mechanisms poses an intrinsic difficulty; on one hand adaptivity requires information; on the other hand the algorithm should be able to function under degraded information. A compromise should be found. Fast adaptivity may require too much information to be feasible. Very slow adaptivity may not be efficient. Deterministic schemes (i.e., slowest adaptivity of all) are very reliable but may be highly inefficient. Controlled Aloha provides a limited adaptivity and maintains the high reliability. Reservation schemes and learning schemes may provide an excellent adaptivity at the price of low reliability and/or enormous control overhead. Let us proceed to develop some possible practical solutions to the problem.

One form of a control subchannel is the acknowledgement traffic. In chapter five we develop a general control principle and a general control mechanism which employs the acknowledgement traffic only. It is also possible to use the acknowledgement traffic to estimate the total number of busy PRUs. The solution of this problem requires that we filter a jump process. The solution has been derived, in conjunction with the problem of controlling the ALOHA scheme, by A. Segall [SEGA76]. The derivation is general enough to apply with minor modifications to the Urn scheme. However, the acknowledgement traffic

may not be a satisfactory information collection mechanism. The information about the state of the system, gathered from the observation of the channel state, does not allow accurate tracking of the state of the system. Only slowly varying statistics of the state may be infered with some accuracy. Adapting to such slow changes may be insufficiently slow and inaccurate for an access scheme. Therefore we shall not pursue the subject further.

Rather, we shall describe a simple information exchange scheme, which offers an accurate estimate for the number of busy PRUs. For the purpose of simplicity we shall restrict ourselves to a one hop system. The problem of a multi-hop network will be considered in a later section.

The Urn scheme requires a multi-access binary erasure channel, which is shared by all PRUs. In order that each PRU may keep track of the number of busy PRUs, it need only be aware of changes in the number of busies. That is a PRU needs to use the control channel only to announce that he became busy or idle. Let us assume that a PRU going idle augments a piggy-back announcement to his last packet. Thus, we shall only be interested in the problem of announcements when an idle PRU turns busy. A newly busy PRU uses the control subchannel to send a standard message (a few bits long). There are three possible announcements that may be received through the control channel: *no new busies* (0); *one PRU became busy* (1); *two or more became busy* (in this case the anouncements collide over the reservation channel and an "erasure" is detected). That is, we use a symmetric reservation channel with collisions.

The information provided by announcements is insufficient to determine the number of busy PRUs precisely. However, as we shall show now, the

announcements over this erasure channel provide an excellent estimate of the number of busy PRUs.

Let us assume that the total rate of arrivals to the system is r. That is $r \triangleq$ expected number of arriving packets per slot; for stability we require $0 \le r \le 1$. Clearly the worst system, as far as acquisition of symmetric information goes, is one where each arrival turns an idle PRU into a busy one. We shall assume that all N PRUs are idle and the arrivals to each PRU form a Bernoulli process with rate r/N. As N grows to infinity (again the worst case) the overall arrival process is Poisson distributed with rate r.

The probability of two or more arrivals during the same slot, is given by:

(2.2-29) $$P[\text{"erasure"}] = 1 - e^{-r} - re^{-r} \le 0.26$$

The probability that three or more packets arrived, given that at least two arrived is:

(2.2-30) $$P[\text{ Over 2 arrivals} \mid \text{"erasure"}] = 1 - r^2/2(e^{-r}-1-r) \le 0.3$$

In Figure 2.2-5 below, we describe this conditional probability (i.e., 2.2-30) as a function of r.

The conditional expected number of arrivals, given "erasure", is given by:

(2.2-31) $$\sum_{k=2}^{\infty}[kr^k/k!] \times [e^{-r}/(1-e^{-r}-re^{-r})] = r[1 + r/(e^{-r}-1-r)] \le 0.24$$

This expected number of arrivals is depicted in Figure 2.2-6 below.

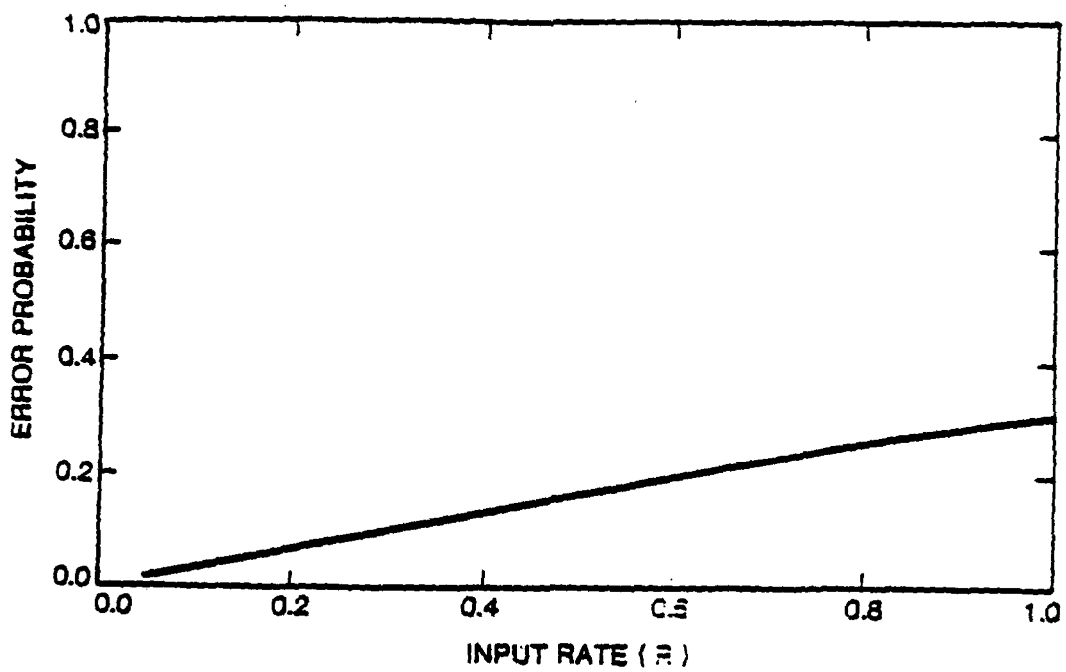

Figure 2.2-5: Probability of estimation error vs. input rate

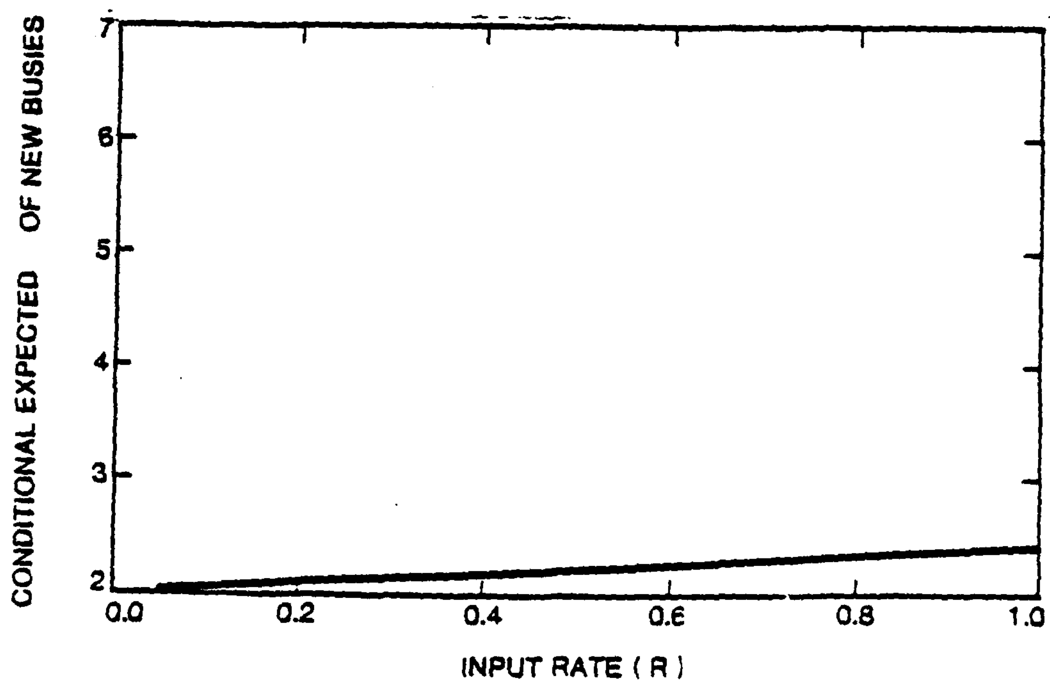

Figure 2.2-6: Expected number of arrivals, conditioned upon "erasure"

The worst case (i.e., infinite idle population*) analysis of error on the control channel, implies a simple estimation strategy when a collision occurs over the

*Note however, that we assumed that the input traffic rate does not exceed the capacity of the channel.

control channel. Indeed, all we have to do is keep an underestimate of the total number of "busies". This is obtained if we decide to interpret erasure as an arrival of two. Erasures are rare (according to equation 2.2-29). The probability that an erasure is underestimated, i.e., more than two PRUs became busy while the estimate is two, is small as shown by equation 2.2-30. An underestimate will occur in the worst case at about seven out of hundred slots. The probability of error is very small even for an arrival rate close to 1. If we choose to have a consistent underestimate, we may correct our estimate every time that our lower estimate shows that only one PRU is busy. In this case k=N and all busy PRUs will transmit and collide. They may use the collision as an indication that there is more than one busy PRU and update their counters to 2.

Still another strategy may measure r and use the conditional expected number of arrivals given by equation 2.2-31 as the data for update. Again, we may correct our estimates every time the system becomes idle.

It is generally better to keep an over-estimate of n rather than an underestimate. The reason is that we may keep the number of transmission rights k too high and enter a state of blocking when the system is heavily loaded. However, it is possible to switch between under-estimates and over-estimates as the traffic load grows. Indeed, there is a wide room for many possible estimation strategies.

Another design parameter of interest is the rate of updates. It is not necessary to update the occupancy counter every single slot. One may choose to update new arrivals once in a few slots. Updating the number of newly turned busy every frame of few slots, requires that we should estimate the number of busies

from both the update information and the expected rate of arrivals.

Finally, we see that a few schemes are available for estimating the number of busies. We use an announcement channel like previous schemes [SCHO77, ROBE73]; however, our control subchannel is very small in comparison with the data channel. We use a fixed amount of channel for announcements, independent of the number of PRUs. We permit collisions over the control channel and do not require coordinated announcements. All these properties make our announcement scheme different and more desirable than previous reservation type schemes.

2.2.3.2 Coordinated Decisions

In the previous sections we saw that the assumption of homogeneous information implies that there is no need for extra coordination. In reality, however, this is only an approximation and it is required to design the access scheme so that it becomes insensitive to perturbations of the conditions under which it was developed. The specific details of the implementation depend upon the particular environment to be considered. However, all implementations of the Urn scheme have to resolve two problems:

1. How many PRUs should be given transmission rights during a given slot.

2. How should the network members agree upon the identity of those to be given transmission rights.

The first problem can be resolved in at least two ways. First we have the solution $k=\lfloor N/n \rfloor$ developed in the previous article and the few methods for

estimating n. Second, we shall develop in chapter five a more general method to decide on k, which does not require any estimate of n.

The second problem may be resolved through some preprogrammed priority mechanism. One possibility is to use a lottery mechanism to generate a fair randomized priority mechanism. To implement random priority mechanism, we equip each PRU with a pseudo random number generator from which he may draw numbers uniformly distributed between 1 and N. The coordination of the decisions of the different PRUs is obtained by using the same seed for the random generator.

At the begining of each slot, each PRU uses the estimate of n, the number of busies, to compute the number $k = \lfloor N/n \rfloor$ of "balls" to be drawn from the urn. He draws k numbers from his random generator. If one of the k numbers turns out to be his own number, the PRU knows that he has a right to transmit a packet. A PRU which is lucky and has a packet to transmit should transmit. There should be an acknowledgement mechanism to detect collisions so that unsuccessful packets will be retransmitted.

There are numerous possible variations of the basic scheme:

- It is possible to update the size of the window k every few slots rather then every single slot.

- It is possible to draw a random permutation of $\{1,...N\}$ and let the next k transmit at each slot until we reach the last PRU. This scheme will permit every PRU a transmission right per cycle. As we shall see, the performance is improved just like the

improvement of round-robin TDMA over random TDMA.

- It is possible to use a window of adaptive size, which rotates in a round robin fashion through the network members, giving them access rights.

Finally, we may use the acknowledgements to acquire an asymmetric information and improve the performance of the Urn scheme. To see how such an improvement may be obtained let us reconsider the urn model.

Let us assume that we just drew k balls out of the urn. If an acknowledgement is instantaneously available then we have in our possession an asymmetric information about the state of the k balls in our hand. There are three possible events that we may detect:

1. A *success*: One black ball and k-1 white balls.

2. *Collision* At least two black balls.

3. *Empty*: All k balls are white.

In the first and the third cases we have increased the information available for us significantly. Our basic scheme does not utilize this information; all k balls are returned to the original urn, there is no learning. An immediate improvement is, in the first case, to let the lucky PRU use the following slots until it empties; in the third case, the obvious improvement is to put the k white balls into a new urn and draw a window of $k'=\lfloor(N-k)/n\rfloor$ from the old urn.

In the second case, we have to decide between drawing from the old urn or

from the k balls in our hand. Again, we introduce a new urn and put the k balls inside. There are two decisions now; from which urn to draw and how many.

In the general case we may keep a number of urns. At each slot we have to make three decisions:

1. From which urn to draw.

2. How many balls to draw.

3. To which urn should we return the balls.

The number of urns represents the memory span of our decision scheme. The more urns we use the less is the value of our symmetric information compared to the acknowledgements. If the number of urns is small we may add a scheme of announcements to acquire symmetric information about each urn. When the number of urns grows to N, we get a perfect information scheme. What is the best number of urns? How to use the urns and the acknowledgements optimally? We do not know.

To summarize, one may use the asymmetric information, generated by the acknowledgements, to improve the performance of the Urn scheme. We can point out few ad-hoc schemes of improvements, but we do not know the optimal solution.

Finally, with a multiaccess erasure reservation channel and a randomized priority mechanism to determine which $k=\lfloor N/n \rfloor$ PRUs have a right over a given slot, the Urn scheme is very robust. It meets the two qualifications of the previous subsection (page 75). Indeed, the number k is relatively insensitive to

small errors in the observation process. Moreover, the Urn scheme permits errors in both estimation and coordination, for it permits collisions and can easily be designed to avoid situations where collisions consume the full capacity. All these properties make the scheme a good practical solution to the problem of adaptive access schemes.

2.2.4 PERFORMANCE ANALYSIS

In this section we analyze the performance of four access schemes in similar environments. We compare the delay-throughput and input-throughput performance of the Urn scheme, Optimally Controlled Slotted ALOHA, TDMA and Perfect Scheduling (the ideal performance bound). Access schemes are, as far as queueing theory is concerned, service mechanisms. To analyze the performance of the the four schemes, we must describe the structure of the queueing mechanism completely. That is, we describe the buffering mechanism and the arrival process.

Henceforth we shall assume that each PRU possesses a buffer which may contain one packet only. This assumption is necessary if we wish to use a Markovian model for the total number of queued packets. That is, the assumption of single-message buffer makes the occupancy configuration B^t (see page 46) a *Markov chain.** The assumption of one-packet buffer enables analysis but it also introduces a blocking phenomenon. Therefore we analyze both the expected delay and the blocking probability.

Another simplifying assumption is made to render the queueing process memoryless. Namely, we consider service mechanisms which are time independent. Our model for Slotted ALOHA assumes that access rights are allocated by coin tossing with time independent probabilities. Our model for Perfect Scheduling assumes that access rights are given to a single busy PRU selected randomly (Using conservation laws, [KLEI76] it is possible to argue that any perfect scheduling scheme generates the same expected delay,

*The case of multi-message buffer does not lend itself to a simple ananlysis since a Markovian state description must include the total number of packets in each queue.

regardless of the priority scheme which is used to allocate access rights). Our model for the Urn scheme assumes that the selection of $k=\lfloor N/n \rfloor$ PRUs for transmission rights, is random and time independent. Similarly we consider random TDMA, i.e., only one PRU is given access right at each slot; the selection is random and time independent. In the last two cases a time dependent (say, round robin) allocation of transmission rights is more efficient, as we shall see in the next section.

We consider two models for arrivals. One model assumes that arrivals are independent Bernoulli processes, i.e. "nature" tosses a coin for each PRU and generates at most a single arrival per slot to that PRU according to the result of the toss. The Bernoulli model is sensitive to the blocking effects. Many packets are blocked even when the system is not fully busy. To reduce the effects of blocking we shall consider a second model. The arrivals are generated from one Poisson process of rate r. New arrivals are distributed among vacant PRUs only. The only blocking effect appears when all PRUs are busy and the whole system can not receive new arrivals. In the case of Bernoulli processes, blocking may occur at each PRU even when his fellows are empty. In the case of a Poisson process, blocking occurs only when there is no room for a new packet in the whole system.

Let N be the overall number of PRUs and n be the number busy PRUs. The probability of i PRUs getting a packet during the next slot is given by:

$$(2.2\text{-}32) \quad a_i^n \triangleq \begin{cases} (r^i / i!)\, e^{-r} & i < N-n \\ 1 - \sum_{j=0}^{N-n-1} (r^j / j!)\, e^{-r} & i = N-n \end{cases}$$

for a Poisson arrival process, and

(2.2-33) $$a_i^n \triangleq \binom{N-n}{i} r^i (1-r)^{N-n-i}$$

for a Bernoulli arrival process.

With these models in mind, the number n^t of busy PRUs at the begining of slot t, becomes a homogeneous Markov chain. The structure of the transitions of n^t is described in Figure 2.2-7.

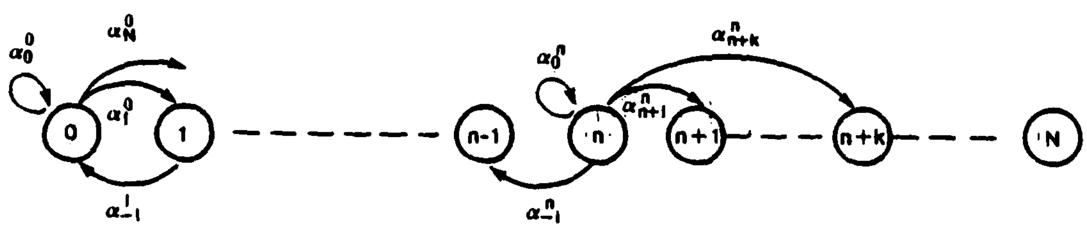

Figure 2.2-7: Transition diagram for the number of busy PRUs

The transition matrix is a lower Hessenberg matrix [WILK]. The steady state distribution of the number of busy PRUs is given by the solution of:

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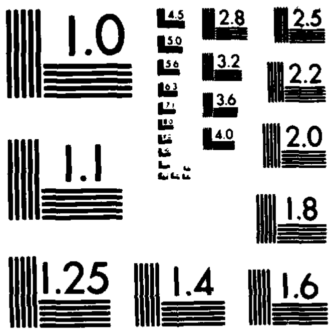

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

$$(2.2\text{-}34)\quad [\pi_0,\pi_1,\ldots,\pi_N] = [\pi_0,\pi_1,\ldots,\pi_N] \begin{bmatrix} \alpha_0^0 & \alpha_1^0 & \cdots & \alpha_N^0 \\ \alpha_{-1}^1 & \alpha_0^1 & \cdots & \alpha_{N-1}^1 \\ 0 & \alpha_{-1}^2 & \cdots & \alpha_{N-2}^2 \\ 0 & 0 & \ddots & \\ \cdot & & \cdots & \\ \cdot & & \cdots & \\ 0 & 0 & \cdots\ \alpha_{-1}^N & \alpha_0^N \end{bmatrix}$$

Where

$$(2.2\text{-}35)\qquad \pi_n \triangleq \text{Probability[n busy PRUs]} = \lim_{t\to\infty} P[n^t = n]$$

and

$$(2.2\text{-}36)\qquad \alpha_i^n \triangleq S(n) \times a_{i+1}^n + [1-S(n)] \times a_i^n$$

Here $S(n)$ is the throughput conditioned on n occupied PRU. Clearly $S(0)=0$ independently of the access scheme. When $n > 0$, $S(n)$ changes from one scheme to another. In the case of a daemon controlled network (i.e. perfect scheduling)

$$S_{\text{PERFECT-SCHEDULING}}(n) = 1$$

Optimally controlled ALOHA provides:

$$S_{\text{OPTIMAL-ALOHA}}(n) = (1 - 1/n)^{n-1}$$

Random TDMA obtains:

$$S_{\text{TDMA}}(n) = n/N$$

Our Random Urn scheme attains:

$$S_{\text{URN-SCHEME}}(n) = \frac{\binom{n}{1}\binom{N-n}{\lfloor N/n\rfloor - 1}}{\binom{N}{\lfloor N/n\rfloor}}$$

The steady state equations 2.2-34 lend themselves to an easy recursive

solution. We can solve and compare the performance of the four different schemes: optimal ALOHA, TDMA, our Urn scheme and a daemonic scheme of perfect channel allocation.

The solutions for the four different schemes are displayed in the following graphs. Figure 2.2-8 depicts the delay-throughput performance of the Urn scheme for Poisson and Bernoulli input rates. The two curves are not radically different. The Bernoulli arrivals curve shows smaller delay for input rates smaller than ~0.6 and larger delay as the input rate increases. This may be easily explained; for low input rates the individual blocking of Bernoulli arrivals reduces the actual input and thus packets which are accepted are served faster. When the load increases the Bernoulli process suffers the same delay but delivers less throughput than the Poisson model. These differences are not critical. Therefore we shall describe the performance of the Poisson model vis-a-vis the four schemes, ignoring the Bernoulli model.

Figure 2.2-9 depicts the delay-throughput performance of the four schemes for 10 PRUs. There is no dramatic change in the relations between the curves as the size of the system grows. Figure 2.2-10 shows the same set of curves when the number of PRUs is 100. The Urn scheme performs comparably to optimally controlled ALOHA when the traffic is very light; it smoothly converges to the performance of TDMA when the traffic grows heavy; in the medium traffic range it is better than both schemes. The distance between the lowest bound and the Urn scheme is the price that we have to pay for not having a daemon to allocate access rights perfectly. Figure 2.2-11 depicts the relation between offered input rate and the throughput. Recall that the only blocking effects

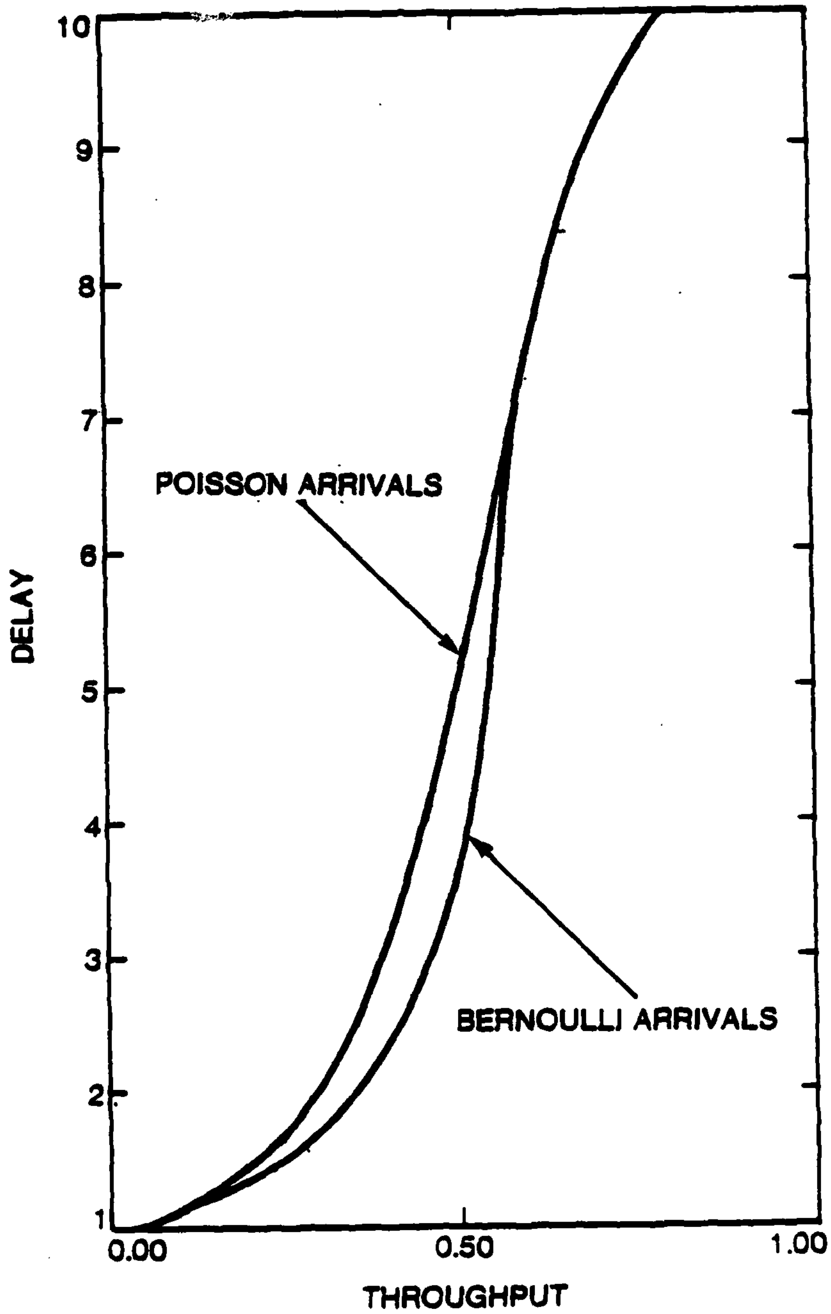

Figure 2.2-8: Delay-Throughput, Poisson vs. Bernoulli arrivals

appear when the full system is busy. The Urn scheme does not limit the useful capacity while optimally controlled ALOHA cannot obtain a throughput beyond a certain threshold which approaches 1/e, as the system size grows, very rapidly.

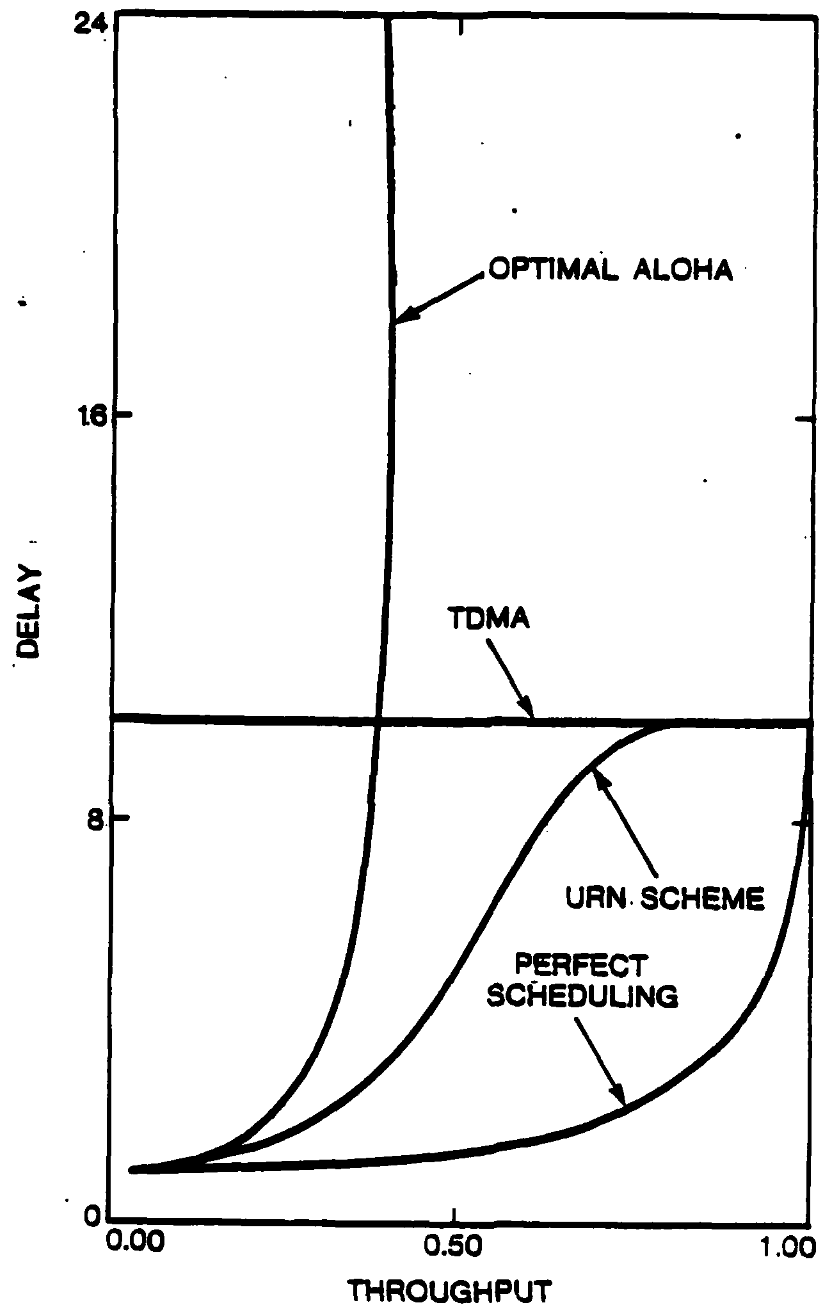

Figure 2.2-9: Delay Throughput performance for N=10 PRUs

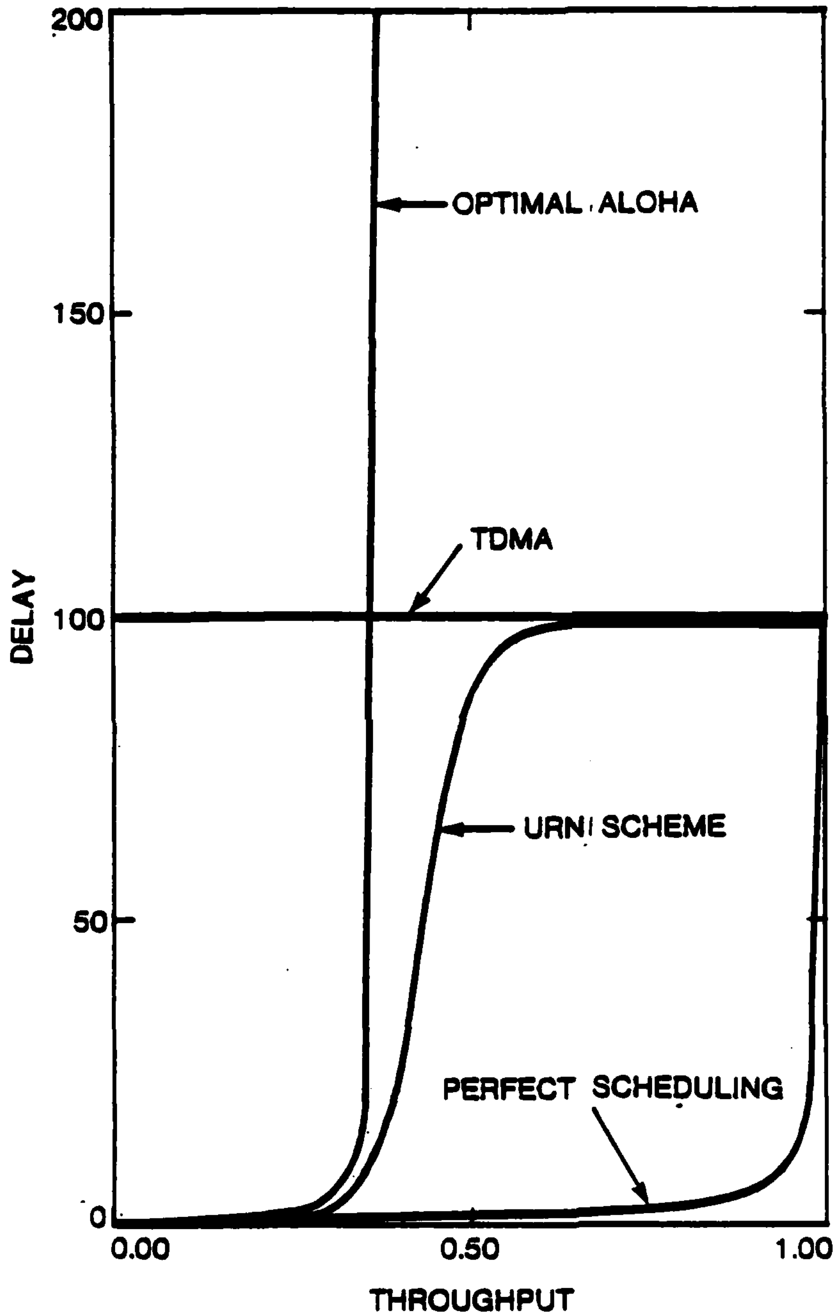

Figure 2.2-10: Delay-Throughput performance for N=100 PRUs

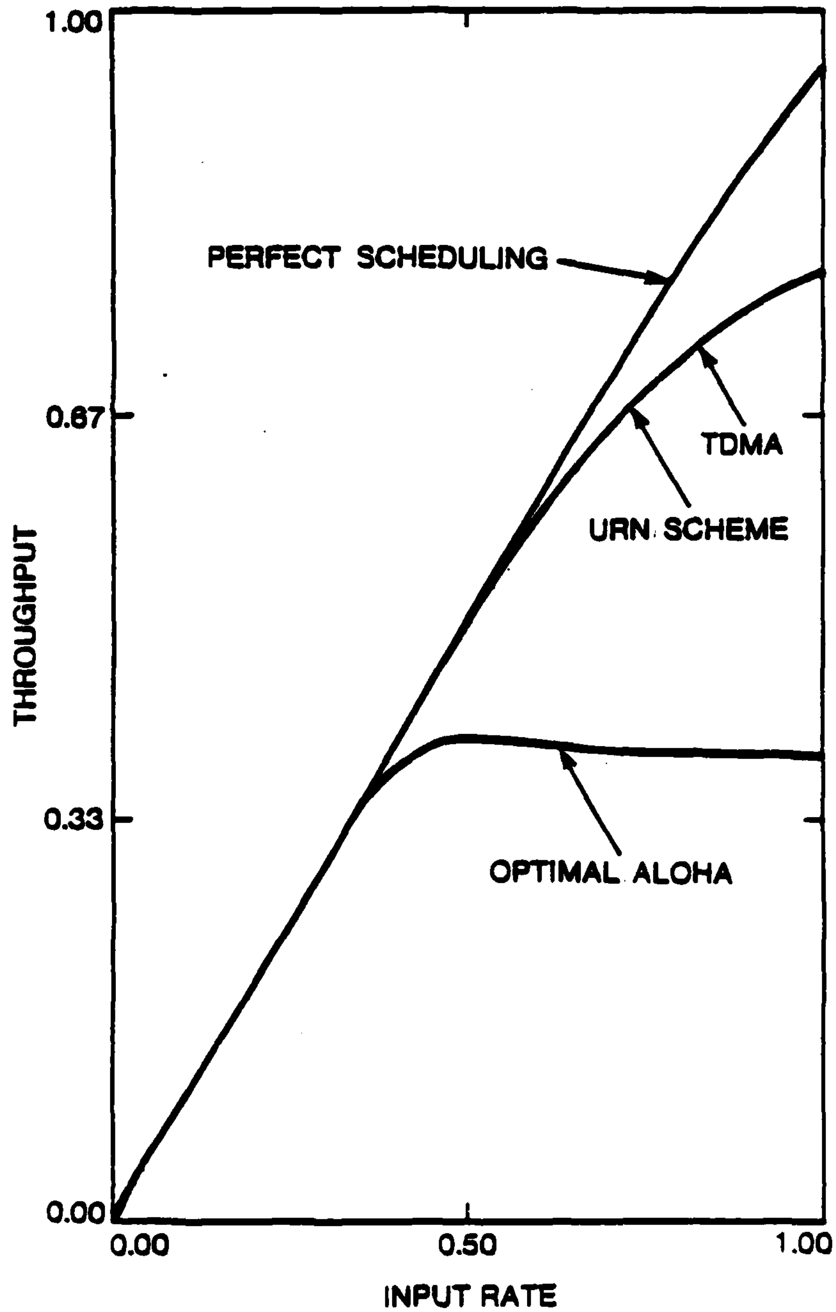

Figure 2.2-11: Throughput vs. offered load

2.2.5 SIMULATION RESULTS

We simulated the different access mechanisms under similar buffering conditions and arrival process. We considered the following schemes:

1. Perfect scheduling

   At each slot a single busy PRU is selected randomly and serviced successfully.

2. Optimal slotted ALOHA

   At each slot, each busy PRU may transmit with probability 1/n, where n is the total number of busy PRUs.

3. TDMA

   At each slot a single PRU is given access rights; if the lucky PRU has a packet ready, it transmits. The selection of the lucky PRU may be random (random TDMA) or pursue a round-robin scheduling (round-robin TDMA).

4. Random Urn scheme

   At each slot $k = \lfloor N/n \rfloor$ PRUs are randomly selected for transmission (N is the total system size).

5. Round-robin Urn scheme

   At each slot $k = \lfloor N/n \rfloor$ PRUs are selected for transmission. The selection is from a random permutation of the numbers {1....N}

until the permutation is exhausted and then a new permutation is drawn and the process is repeated. Thus, each PRU is guaranteed access rights once in N slots.

6. Window scheme

Here we incorporate learning from acknowledgments into the Urn scheme. At each slot we let a rotating window point to owners of access rights. The size of the window is originally $k=\lfloor N/n \rfloor$. If a "collision" occurs the window is stopped and its size is divided by 2 (i.e., the upper half of the older window is removed). If a "success" or "empty" is recorded, the window is rotated to the end of the previous window and it's size is reset according to the new value of n (i.e., $k=\lfloor N/n \rfloor$). One can use other variations of the Window scheme, incorporating more sophistication into the movement of the window.

Figure 2.2-12 depicts the delay-throughput performance of the first four schemes, for a system with N=10 PRUs each possessing a buffer for 25 packets. Figure 2.2-13 depicts the respective input-throughput performance.

Figure 2.2-14 and 2.2-15 compare the results of analysis to measurements from simulation. We consider the Random Urn scheme for a system with N=10 PRUs. The simulation results show a striking match to those obtained from analysis.

In figures 2.2-16 and 2.2-17 we compare the performance of the Random Urn scheme, the Round Robin Urn scheme and the Window scheme. The Round

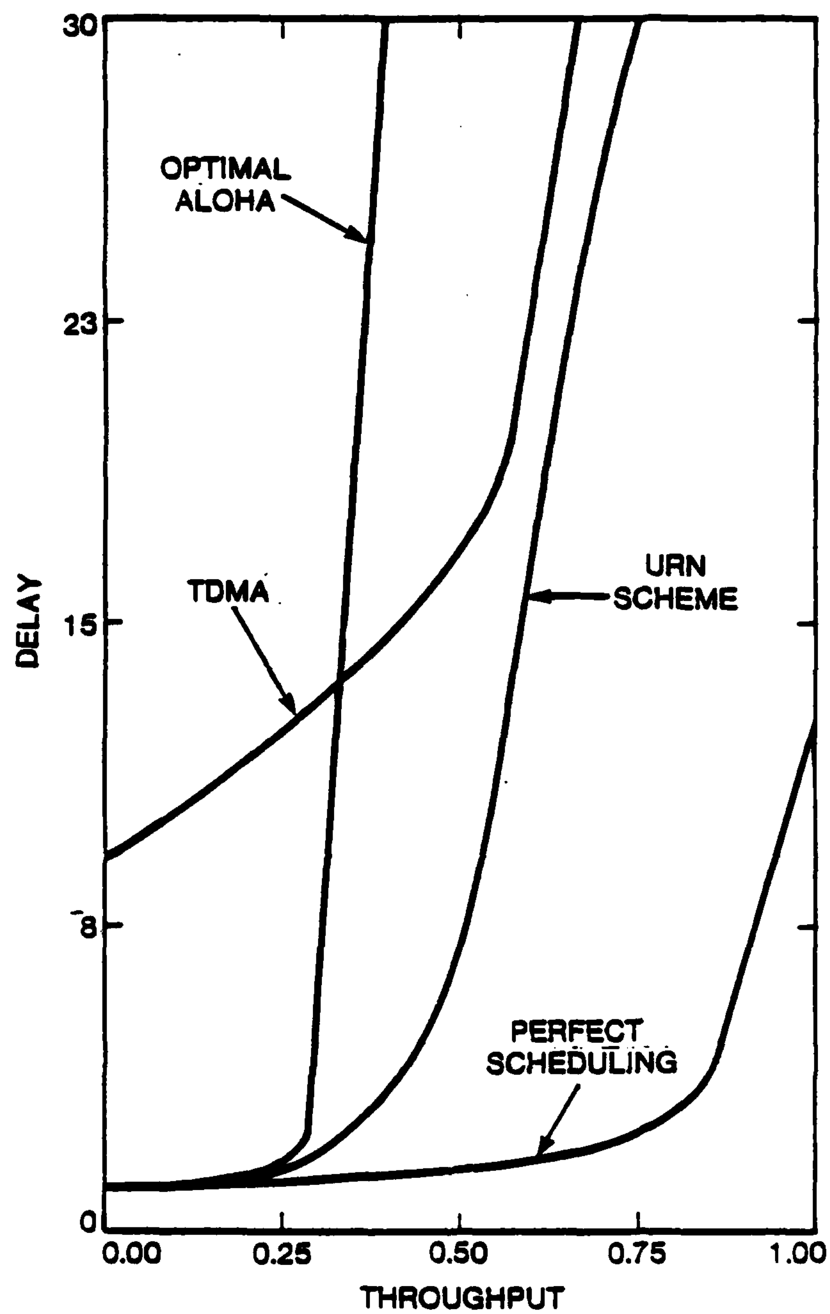

Figure 2.2-12: Delay-Throughput performance for N=10 buffered PRUs

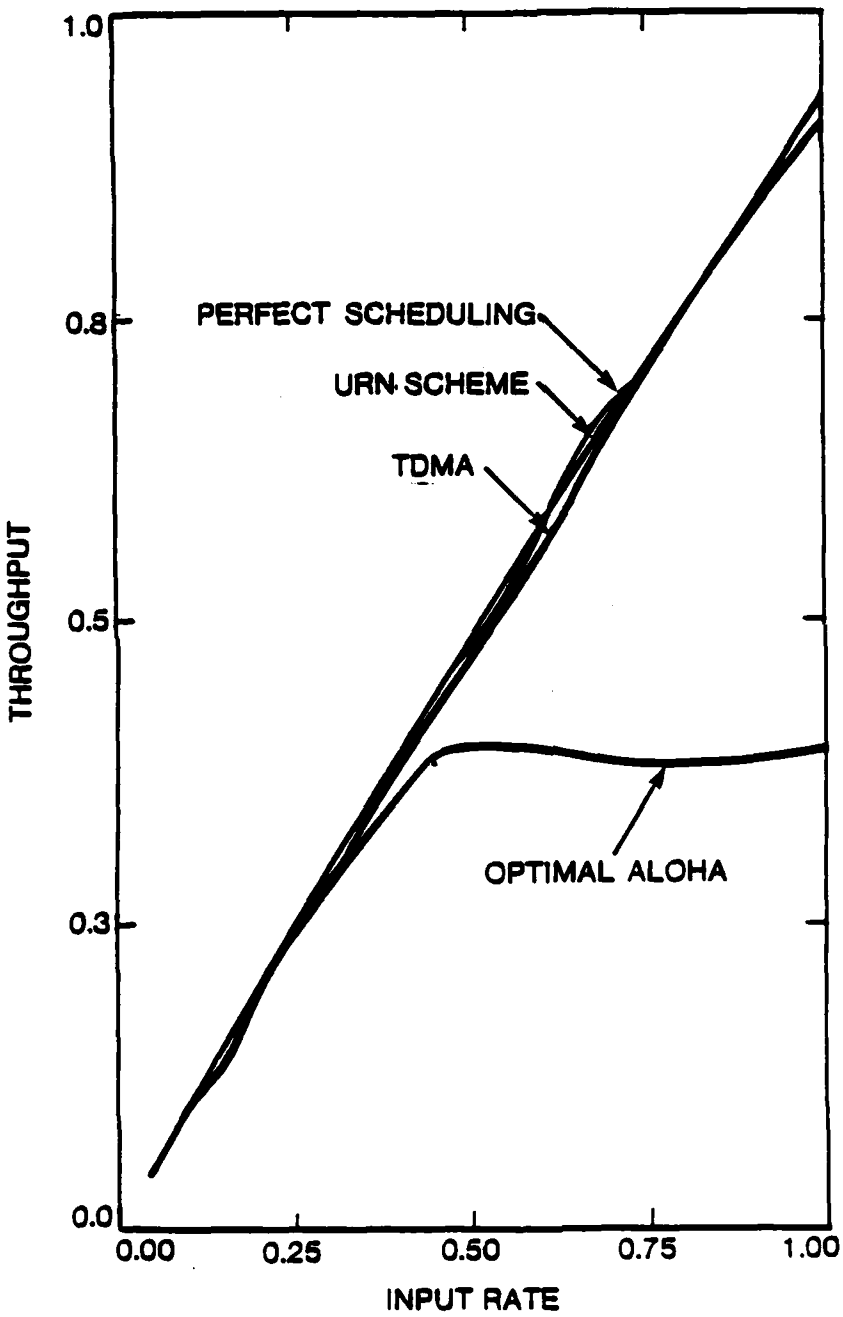

Figure 2.2-13: Throughput vs. offered load for N=10 buffered PRUs

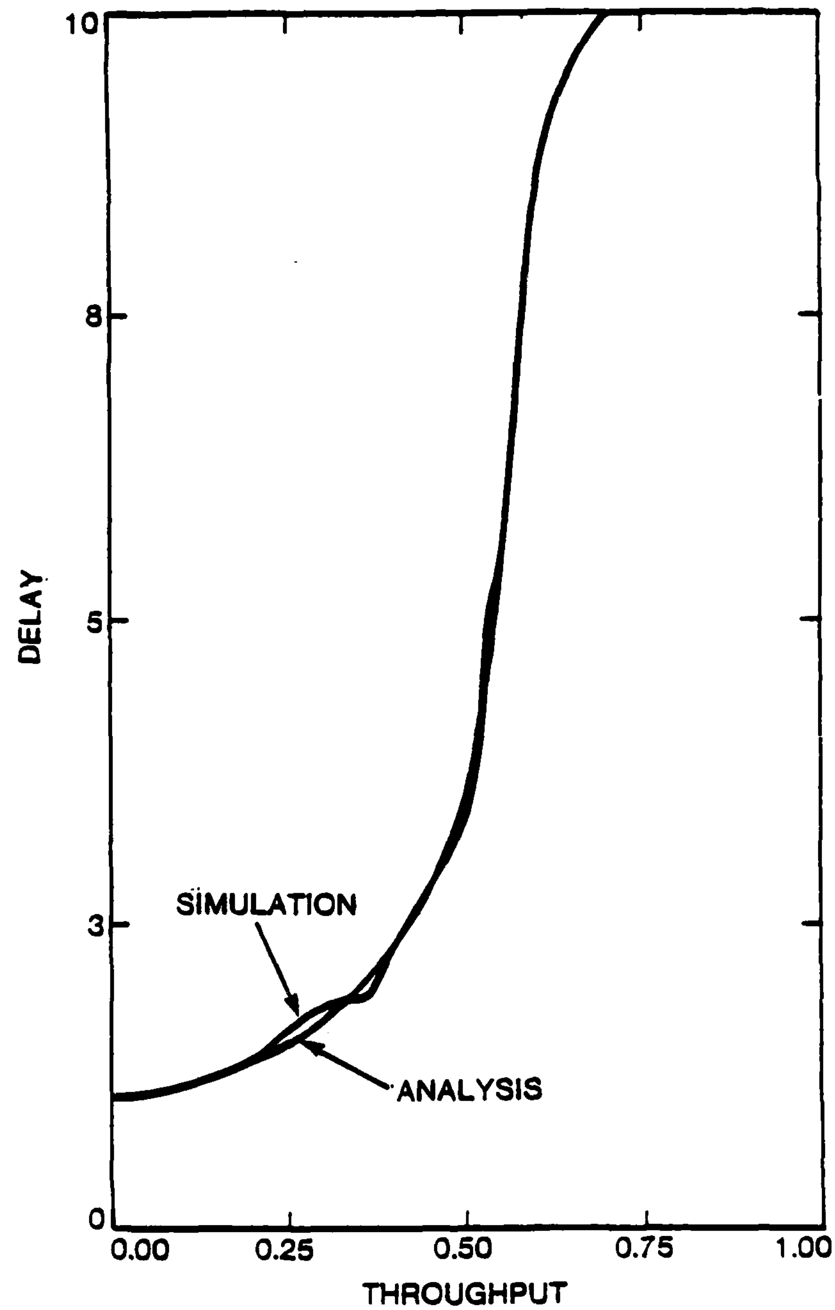

Figure 2.2-14: Delay-Throughput performance -- analysis vs. simulation

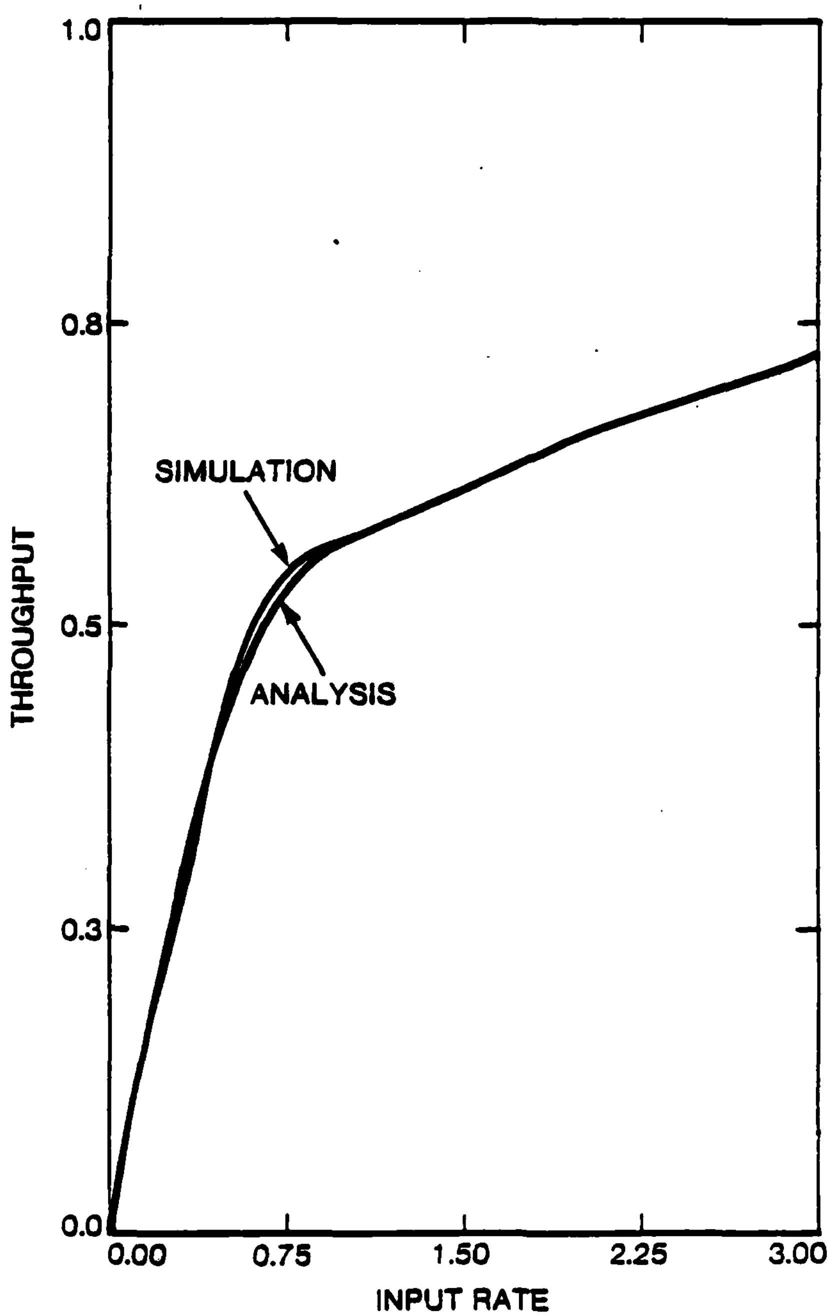

Figure 2.2-15: Throughput-offered load -- analysis vs. simulation

Robin Urn scheme scheme shows an improvement over the Random Urn scheme when the traffic is in the medium range. The Window scheme provides an additional improvement which could only justify the increased complexity when the traffic is heavy.

Finally, figure 2.2-18 illustrates the power of our multipleaccess erasure reservation channel. The two curves represent an a-typical deviation of the estimated number of busy PRUs from the actual number of busy PRUs (using underestimates). To produce even this slight deviation, we had to condition the system to a very heavy traffic (input rate 0.8 packets per slot) and sample an unusually lengthy busy period. Typical curves hardly show any deviation.

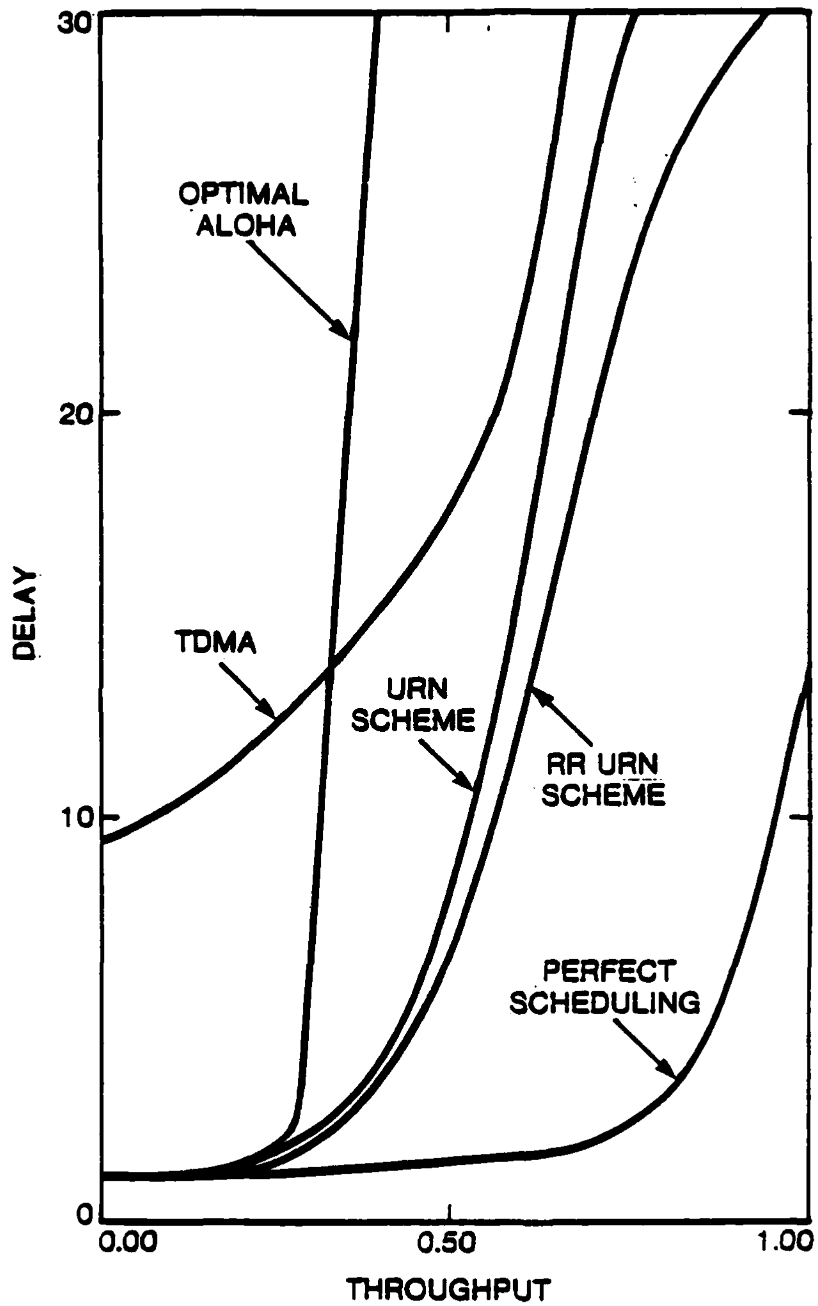

Figure 2.2-16: Delay-Throughput performance of Urn scheme implementations

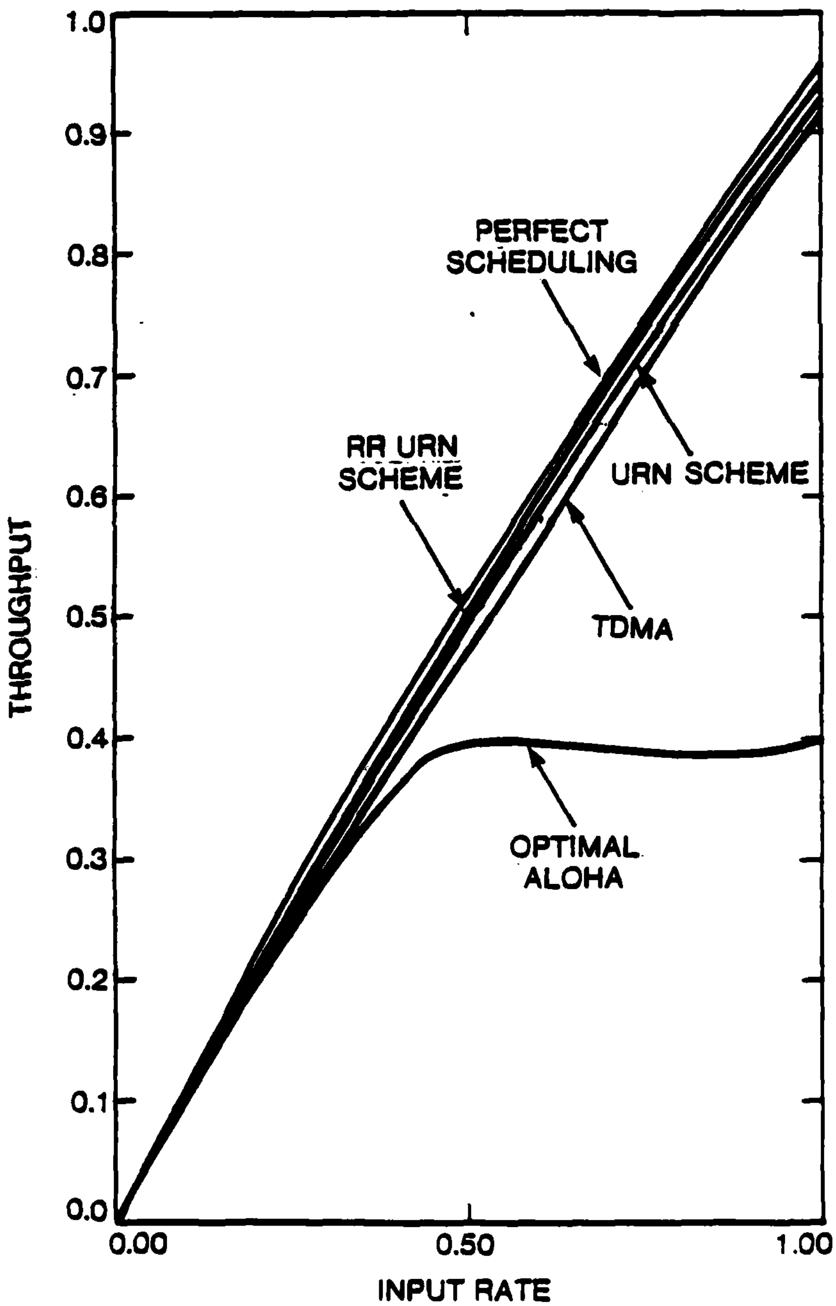

Figure 2.2-17: Throughput-load performance of Urn scheme implementations

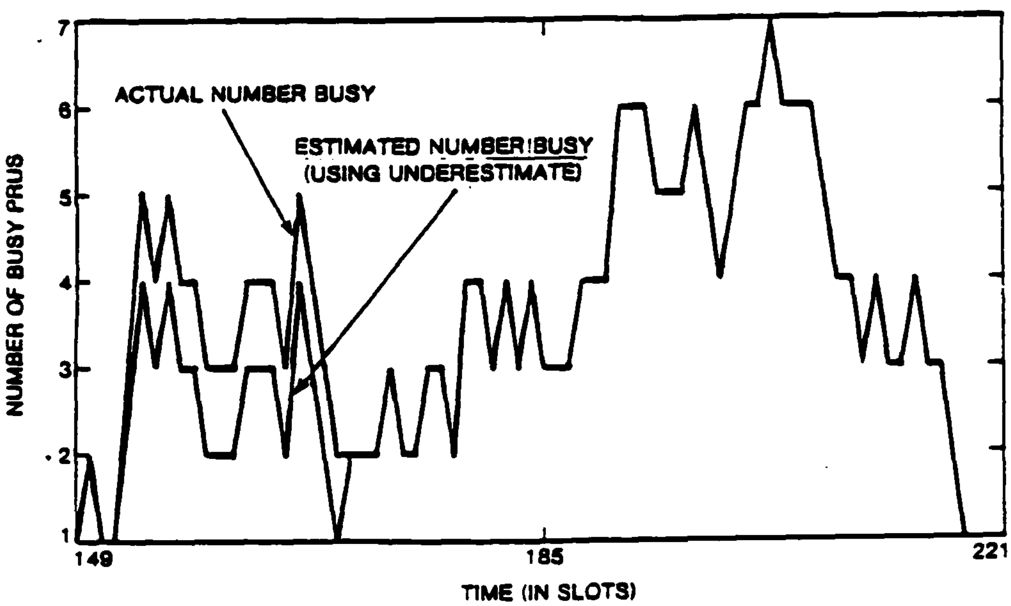

Figure 2.2-18: A rare deviation of estimated number of "busies" from actual

2.3 FROM ONE HOP TO MULTI-HOP NETWORKS

2.3.1 SOME OBSERVATIONS ABOUT MULTI HOP NETWORKS

The problem of a multi-hop PRNET organization is very difficult, as has already been noted in the first section. Resolving the conflict between simultaneous demands for communication leads to hard combinatorial problems. Reservation schemes, polling schemes, or any other class of completely centralized adaptive schemes are impractical, due to the complexity of the scheduling problem. On the other hand, pre-determined allocation schemes which do not adapt to the variable communication demands, are highly inefficient. Again we would like to have an adaptive scheme, which can be implemented as a decentralized decision mechanism with a small overhead.

In what follows we consider only tree-like networks, that is networks where all traffic is directed towards a central station (note: only the routing graph is a tree, not the hearing graph). Figure 2.3-1 depicts a typical such network. Tree-like networks possess a natural layer structure. The first layer consists of all the PRUs that are heard by the station; the second layer consists of all the PRUs which are heard by the first layer but not by the station, and so on.

Let us observe some properties of multi-hop tree networks:

1. In a multi-hop PRNET the channel is distributed among the different PRUs; each PRU possessing a local copy of the channel. Therfore a new dimension, i.e., space, is added to the two dimensions of the single-hop channel (i.e., time and bandwidth). Since each PRU shares the "local" channel at each one of his

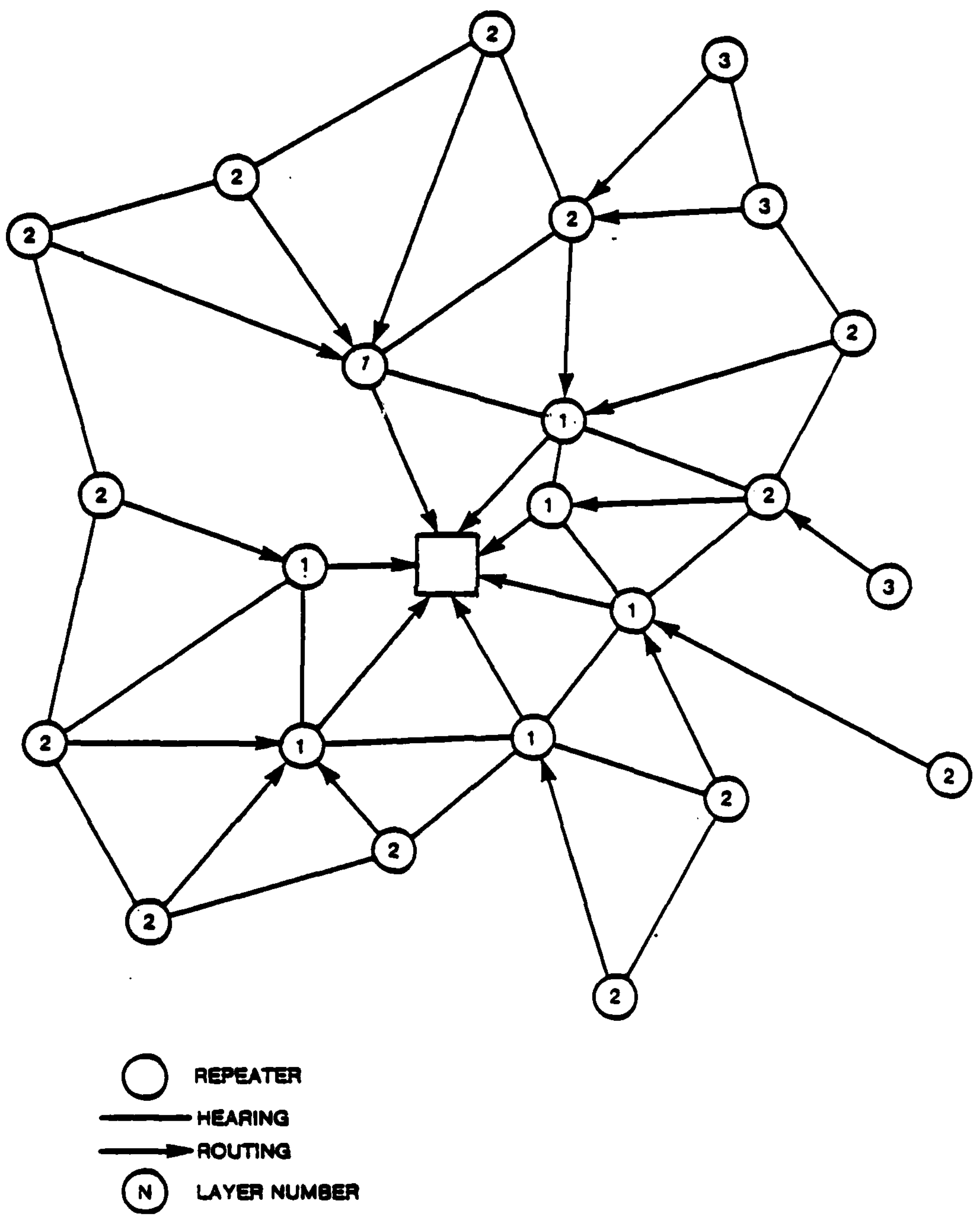

Figure 2.3-1: Layered structure of a tree-like PRNET

hearing neighbors, the problem of resolving conflict of use becomes a problem of a simultaneous conflict resolution at each one of his hearing neighbors.

2. The passage of a packet through a PRU requires at least three slots of it's channel. The first slot for arrival, the second slot for transmission and the third slot when the packet is heard again, once it is relayed farther (i.e., the "echo"). Thus, at most one third of the channel may be used at each PRU (with the exception of the first hop and the station).

3. A multi-hop PRNET is channel bound and not buffer bound; the congestion of the channel dominates the scene. This is an important experimental (simulation) and analytical result of F. Tobagi [TOBAGI77]. The number of buffers with which we equip PRUs does not influence the expected delay of packets in any significant manner. On the other hand, a small change in the hearing topology, the organization of the channel etc... may influence the expected delay significantly.

4. Bursty, multi-hop traffic is better served by a cut-through* mechanism than by a store and forward mechanism [KERM77].

These observations will serve as a basis for the considerations of multi-hop network organization to follow.

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*"Cut-through" denotes a mechanism where an arriving message will be immediately repeated if possible, and if not, it is stored and forwarded later.

2.3.2 DECOMPOSITION OF TREE-LIKE NETWORKS

Let us consider the problem of conflict resolution from the point of view of an individual network member. When he decides to talk, he may have a conflict of use with other members which try to use the channels occupied by him (i.e., local channels at all his hearing neighbors). Thus, our designated PRU has to coordinate his decision with all network members with which he may interfere. Similarly, he has to coordinate his decision with all the network members which interfere with him. Our PRU faces a formidable problem of organization.

The layer structure of tree-like networks renders the problem slightly easier. Indeed, a PRU in the n-th layer can only interfere with PRUs in his layer and the two successive layers (i.e., n+1-th and n+2-th layers). Therefore, conflict resolution is localized to a smaller environment. Nevertheless, the problem of coordinating decentralized decisions even in these smaller environments, is still very difficult.

It seems that an implementable, adaptive algorithm is impossible, due to the size of the environment to which each PRU needs to adapt. This is clearly a "curse" of the combinatorics of interference. In this section we examine the possibility of decomposition, as a solution to the combinatorial complexity of coordination.

Decomposition is a handy approach to large-scale system organization. We shall describe a few methods through which our problem may be decomposed into simpler problems. The tree-like organization of the network already obtained a substantial decomposition of the interference graph. Further

decomposition is obtained if we decide to ignore "backward" interference. That is, a PRU has only to resolve conflict of use with members of his layer, but not with members of successive layers. Such choice of policy we call "rude".

The idea is simple: a PRU should give up his access right only to avoid the danger of "blocking". PRUs in higher layers do not endanger our designated PRU for they can not block his transmissions. A "polite" choice of policy (i.e., to give up access right in favor of PRUs from higher layers) is reasonable iff a cross-layer coordination of access rights can be achieved. In the absence of precise information and coordination, politeness makes sense only in an average manner to be discussed in chapter five. To sum up, rudeness represents a choice of a local "max-min" policy in the absence of a global mechanism for a coordinated optimization. Rudeness has an additional advantage: it introduces a natural flow-control mechanism. Namely, new packets are prevented from advancing into lower layers by packets which already reached those layers. Therefore, the flow of packets to regions which are heavily loaded is restricted by the very load itself.

Another method to decompose the conflict resolution problem and restrict it to layers, is to split the channel between layers so as to eliminate inter-layer conflicts. Let us reconsider the network of Figure 2.3-2. We split the channel into 3 different subchannels using frequency or time division. Each subchannel may be considered as a different "color". We assign a transmission channel (color) to each layer in the network. The first layer is assigned a "green" color; the second layer is assigned a "red" color; the third layer is assigned a "black" color; successive layers are colored green, red, black,

alternatingly. Each PRU may use one color for transmission and the other two colors for reception.

Note that with directional antennas only two colors would be required.

Frequency (or Time) division among layers does not pose the same tradeoffs as in a single-hop FDMA. In a single-hop PRNET FDMA (TDMA) preallocate the channel regardless of the immediate channel demand, thus wasting a substantial fraction of the channel and creating undully delays, when the demands are bursty. In a tree-like network, a packet crossing a layer uses the channel at the point of crossing three slots. Therefore, at most one third of the channel can be used. Thus, Frequency Division among layers, using 3 divisions, is only a natural mechanism to render a more orderly crossing between layers. Delays are not made longer and no additional channel wastes are introduced. Frequency Division between layers is, thus, a natural mechanism to obtain effective channel utilization.

Now that our network is decomposed, the competition and conflict resolution are restricted to single layers only. By adjusting his power properly, a PRU may reduce the number of PRUs in his destination layer, (precedeing layer) that hear him, to one or two. The interference may be greatly reduced and adaptive sharing becomes a feasible solution for inner layer interference.

Let us consider a generic network member PR_i. The set of all PRUs from the layer above PR_i, which are heard by PR_i, will be called: *control environment* of PR_i. Each PRU needs only to resolve conflicts within the specific control environment to which he belongs. We make the assumption that a PRU may

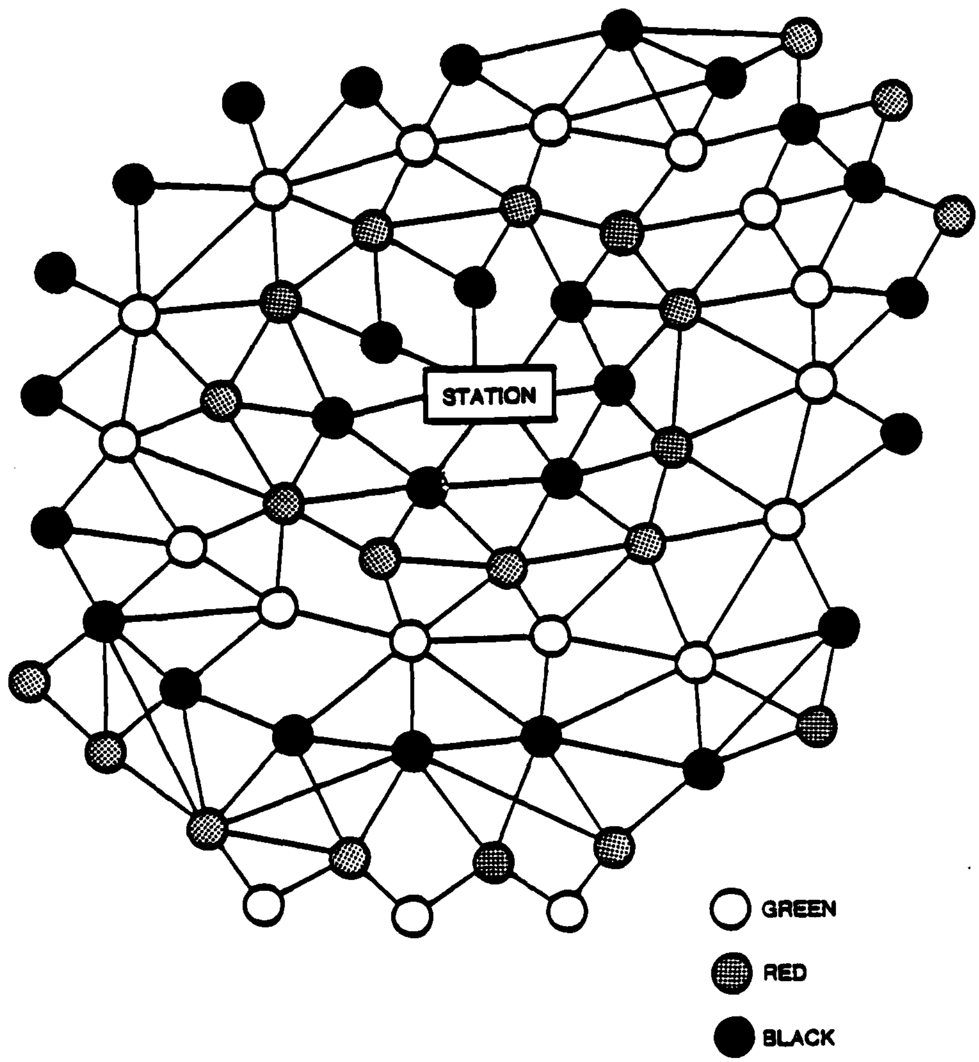

Figure 2.3-2: Decomposition of a PRNET using three channel divisions

adjust his power so that he belongs to at most two environments (the case where a PRU is permitted to be a member of three environments, follows suit).

In terms of the urn model of the previous section, each PRU is a ball which may exist simultaneously in at most two different urns; such a PRU is said to be a "shared" PRU. Each control environment may conduct its own distributed lottery in order to decide access rights to its members (using adaptive coins in an ALOHA manner or drawing from the urn according to the Urn scheme of the previous section). The problem is that shared PRUs have to adapt to two environments. Indeed, shared PRUs are using two lotteries to decide their access rights. They should have no problem of decision if they win or lose in both lotteries simultaneously. However, if they win one lottery and lose the other what should they do?

While an optimal solution of the dilemma of shared PRUs may be complex, suboptimal solutions may be easily described. One may choose to let a shared PRU transmit only when he wins both lotteries. This provides a natural penalty for PRUs which choose to live in two environments (thus causing more interference). The other extreme is to favor shared PRUs letting them transmit whenever they win any lottery. Between the two extremes we have a spectrum of strategies: individual coin tossing to resolve the dilemma; estimating individual probability of success vs. individual probability of collision; etc...

To conclude the discussion we see that decomposition of the access right conflict into small environments, enables us to turn problems of multi-hop organization into problems of one hop organization. In particular, with the aid

of decomposition, our Urn scheme may be implemented in a tree-like network. The experimental and analytical study of these options is beyond the scope of this work and is left for future research.

2.3.3 "CUTTING THROUGH" THE NETWORK

With the aid of decomposition through channel "coloring", we may introduce "cut throughs", i.e., immediate repeating of packets. Since successive layers talk and listen over noninterfering channels, we may use the PRUs as radio repeaters to relay the packets farther as they arrive. This is possible if the PRU is endowed with the capability for a simultaneous use of the three channels. Cut through is also possible in cable networks using a broadcast scheme of communication.

Cut throughs in packet switched networks, have been studied by P. Kermani [KERM77]. The exact analysis of the performance of cut throughs, where collisions occur, is beyond the scope of our discussion.

The distinctive nature of cut throughs in a broadcast network, is that the PRU may serve as an *intelligent channel*. Indeed, suppose a PRU may decide whether the recieved signal is the noise of a collision or a successful transmission, then collisions may be eliminated on their way. The network behaves like a one hop PRNET with an "intelligent" channel which identifies and eliminates some collisions on their way.

As an example of the use of intelligent channels, consider a binary network, i.e., each PRU has at most two "hearing sons" in the successive layer. Let us assume that each busy node decides whether to transmit or not, by tossing a biased coin. A transmitted packet is relayed down the tree from its origin to the root. If two packets reach the same node at the same time the collision is detected and is not relayed forward.

Clearly, the station (root) will receive one successful packet whenever an odd number of packets get transmitted. Whenever an even number of packets are transmitted a collisions occur which eliminate all packets. If p is the probability of transmission assigned to the coins, then the probability of a successful transmission, given that n node are busy, is:

$$(2.3\text{-}1) \quad S = \sum_{j=0}^{\lfloor n/2 \rfloor} \binom{n}{2j+1} p^{2j+1}(1-p)^{n-2j-1} = (1/2)\left[1-(1-2p)^{n}\right]$$

This expression is clearly maximized when $p=1/2$, the maximal value being 1/2 (for any $n>0$). Thus, a binary intelligent channel may use an ALOHA cut through scheme to obtain a throughput of 1/2, with no problems of instability or control. Moreover, the transmission policy and the probability of success do not depend upon the load on the system or its size.

The above example is unrealistic (at least in the context of present PRNET design) because the hearing topology is not a design parameter. It could only serve to demonstrate the value of intelligent distributed channels. Nevertheless, in systems for which the communication channel is an expensive commodity while processing is relatively cheap, one could see the advantage of intelligent channels.

3. *PROBLEMS OF INTERFERING QUEUEING PROCESSES*

(OR, HOW STEEP IS THE ASCENT FROM ONE TO TWO?)

In this chapter we consider the queueing processes occurring in the buffers of two interfering PRUs. This is a typical instance of a multi-dimensional queueing problem in discrete time, that is, a problem of interacting queueing processes. Problems of multi-dimensional queueing processes arise in any computer communication network. Interaction between the different queueing processes arises through the communication protocol (which conditions the activities of one process on the state of the others) and/or the shared communication medium.

In some cases the communication protocol or the communication medium eliminates the dependencies between the queueing processes, and then the multi-dimensional problem may be reduced to a collection of one-dimensional queues. However, if the queues interact properly, through sharing of server and/or arrival processes, it is usually impossible to reduce the dimensionality. Moreover, if an approximate reduction is used, the effects of sharing may be lost and the analysis may miss its very purpose. Therefore the problem of analyzing interfering queues is of prime importance.

When it comes to a problem of a single queueing process there exists an abundance of analytic solutions. Yet, the ideas upon which most solutions rest, may be traced to a common algebraic process [KING63]. It is possible to show (ibid) that multi-dimensional queueing process cannot be solved (in general) through the same methods. Therefore a set of new tools is required.

This chapter presents the problem of two interfering PRUs and some possible approximate solutions. The next chapter is devoted to the development of a new set of tools to solve multi-dimensional queueing processes.

3.1 THE TWO BUFFERED PRUS

3.1.1 THE SYSTEM

We consider two PRUs communicating packets to a common destination over a time slotted shared channel. Packets arrive at PR_i from a Bernoulli source of rate λ_i $(i=1,2)$. The PRUs use a Slotted- ALOHA channel access scheme. That is, a busy PRU decides indepemdently whether it should transmit or not by tossing a biased coin; he transmits if his coin shows Heads*. μ_i $(i=1,2)$ will designate the probability of Heads on PR_i's coin.

Interaction between two PRUs, as those described above, may arise in numerous ways. First the arrivals of packets to a PRU may or may not depend upon his transmissions; dependence arises if the PRU serves as a repeater, thus new arrivals are blocked by transmissions; independence occurs if the PRU is a terminal whose packet production is not interfered with by the communication process. Second, the service process of packets at one PRU may depend upon the service process of the second PRU; e.g., collisions of packets result in a possible loss of service. Third, it is possible for the transmissions of one PRU to interfere with packet arrivals to his fellow. Therefore it is possible to consider many models for "two buffered PRUs" problems.

We choose to consider a few models, increasing the interaction between the two PRUs gradually. We make the assumption that collisions result in a total loss. The difference between the models is in the structure of interaction

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*Note that our model of Slotted-ALOHA does not distinguish between "new" and "retransmitted" packets.

between the two PRUs; this is illustrated in Figure 3.1-1.

1. The first model is that of two terminals talking to a common destination. Interaction is limited to collisions between simultaneously transmitted packets. There is no interaction between packets arrivals and transmissions. This is the classical Slotted ALOHA model with N=2 users.

2. The second model accounts for the case of two repeaters. In addition to collisions between simultaneous transmissions, there is interaction between arrivals and transmissions. Arrivals of packets to each repeater are blocked by his own transmissions.

3. The third model assumes that the two repeaters hear each other. Therefore arrivals to one repeater are blocked not only by his own transmissions, but also by the transmissions of his fellow.

4. The fourth model is a "maximum interference" model. The additional interaction is between the arrivals to the two PRUs. We assume that the population of terminals which generate packets for each PRU is heard by both PRUs. Therefore simultaneous arrivals are precluded by collisions.

It is also possible to consider other models of interaction between two PRUs. For instance, one may consider tandem arrangements, capture effects, flow control mechanisms etc... However, we shall restrict ourselves to the above models only. The methods that we employ may be easily adapted to other

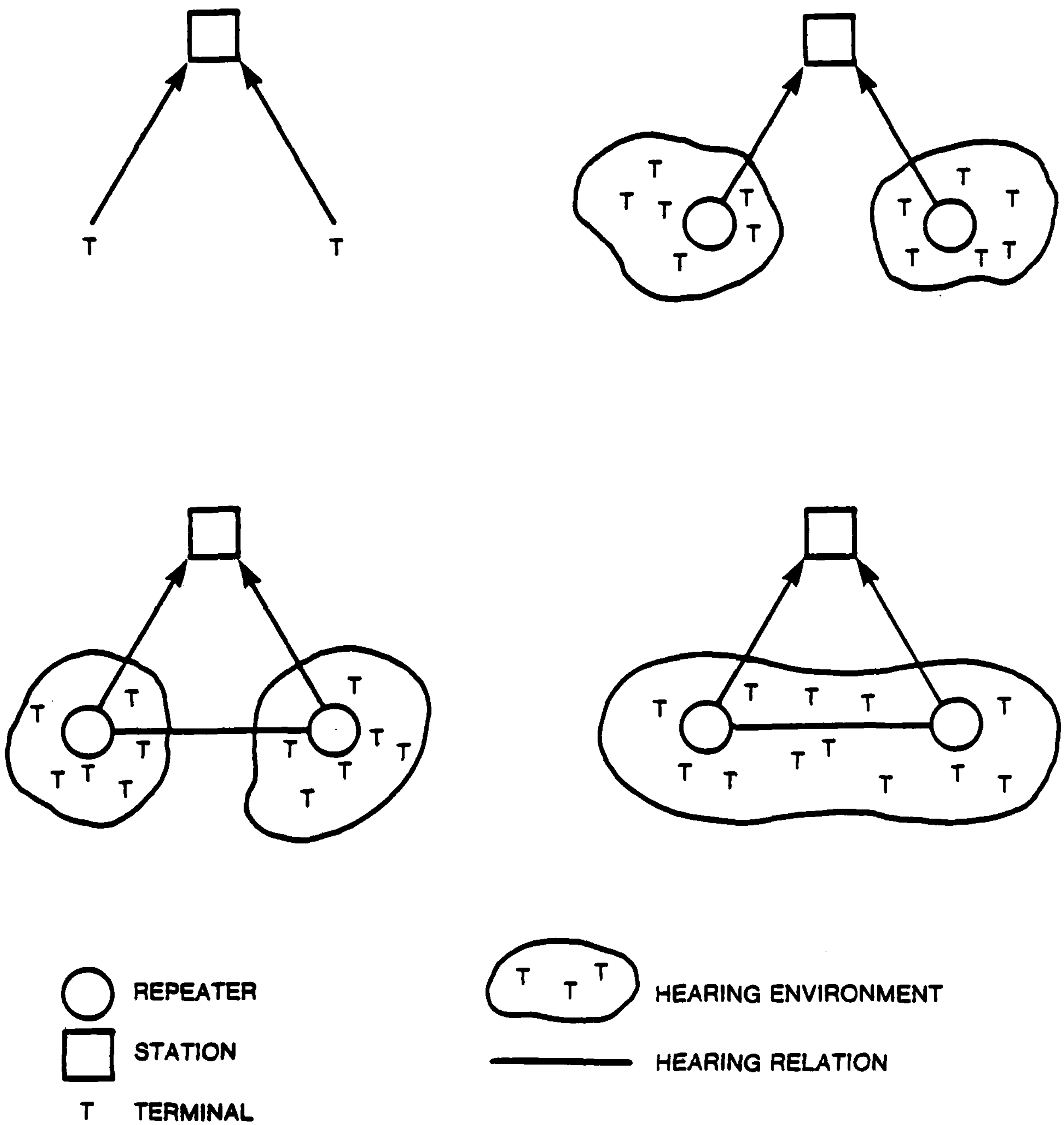


Figure 3.1-1: Four models of interference

models of interaction.

Let Q_i^t be the number of packets buffered at PR_i at the begining of slot t. $Q^t=(Q_1^t,Q_2^t)$ is a random walk (RW) on the positive quadrant of the two dimensional integer lattice. Figure 3.1-2(I-IV) depicts the transition structure of Q^t for our four models of interaction respectively. All four models form a nearest neighbor RW.

Our problem is *to find a closed-form expression for the steady-state distribution of* Q^t *- provided that it exists - in terms of the parameters of the two dimensional random walk (TDRW),* λ_i *and* μ_i. We should also determine the conditions on the parameters so that the RW converges to a steady-state. Both problems are open and hard for, as we shall prove, none of the methods which are used to solve the one dimensional RW in the presence of boundaries, provides sufficient tools to solve our problem.

Problems of interacting queues arise in almost any scheme of resource sharing. Dynamic schemes of sharing present two theoretical difficulties. First, the coupling of queues makes the problem essentially two-dimensional rather than simply a set of independent one-dimensional problems. Second, the change in the nature of the service when one queue (or more) empties presents a difficult boundary value problem.

The combination of dependence and boundary problems is the source of both the theoretical difficulty and the practical value. Approximate models avoid the difficulty, therefore they are usually unsuitable to explore the effects of sharing. Exact solutions, or a system of more refined approximations is

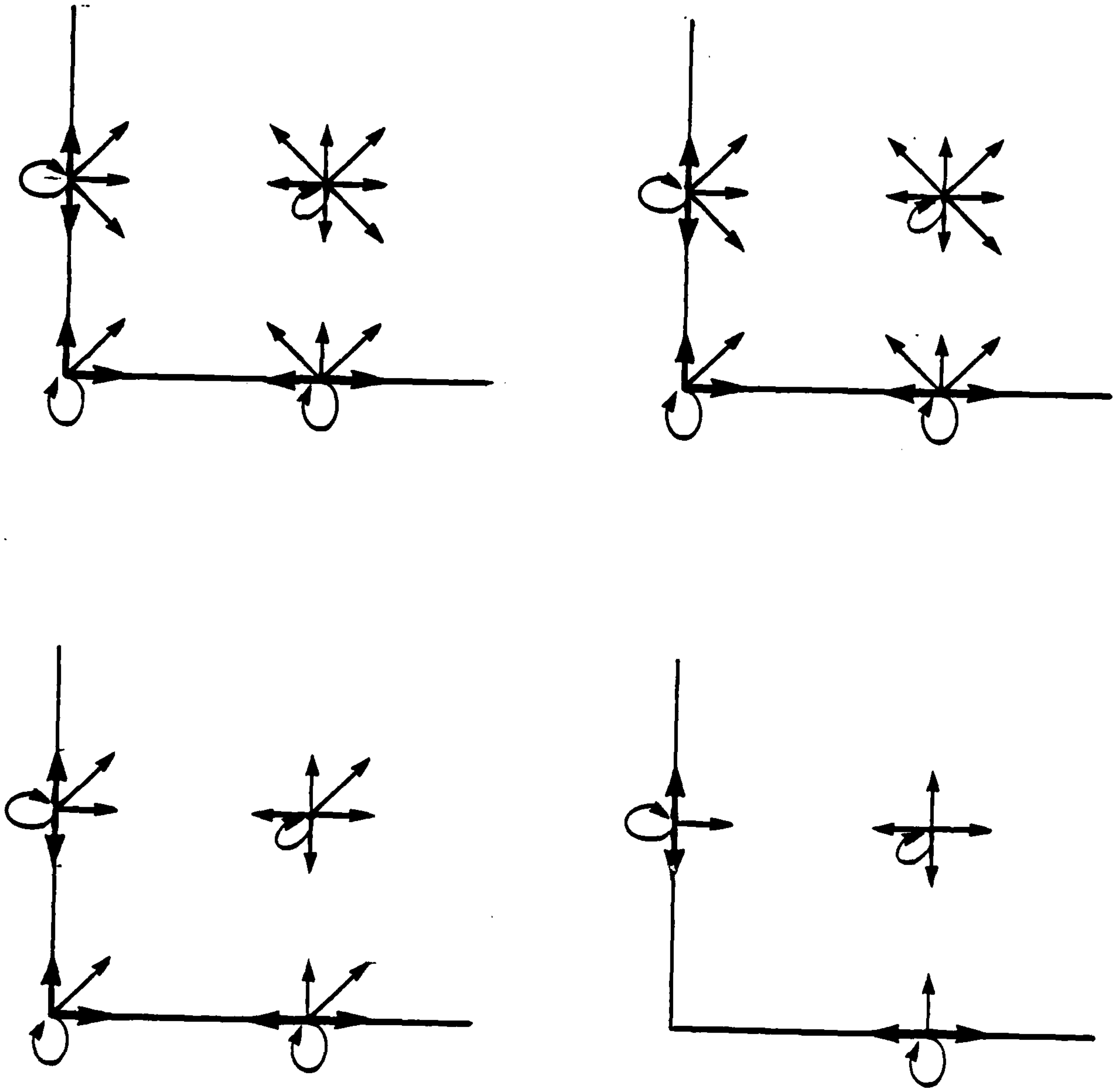


Figure 3.1-2: Transition diagrams for the four interference models

required, if the effects of sharing are to be present in the solution. Therefore, the solution of the two buffered PRUs bears a significance to many other problems of sharing. This significance is reflected in the apparent similarity between the equations for the two buffered PRUs, problems of processor sharing, problems of routing along the shortest queue etc...

The investigation of two dimensional models of interfering queueing processes is the subject of the present chapter and its successor. We proceed with a brief sketch of the history of multi-dimensional RW (MDRW). Then we use Kingman's unifying view of queuing theories [KING63] to show that the inherent difficulty is of the same nature as that of solving G/G/k systems, which is the reason why none of the known methods of queueing theory can help us. This is followed by a development of some simple heavy and low traffic approximation schemes. Unfortunately these schemes are crude and insensitive to the details of the interaction between the two queues. Next we explore our fourth model. Luckily enough, the problem possesses a product form solution and may be completely solved within the scope of simple methods. The solution has some surprising features, foremost among which is a singular improvement of performance when the PRUs choose to be rude, i.e., transmit with probability $\mu_i=1$. Moreover the increase in interaction, from one model to another, beyond a certain threshold seems to have only minor effects upon the behavior of the system. Therefore the delay- throughput performance of the fourth model can serve as an approximation for the first three models. This approximation is excellent for the second and third models as shown by simulation. Finally we use Kingman's stability conditions to solve the stability problem deriving capacity results.

It is impossible to apply the simple methods of this chapter to obtain exact solutions for the first three models. When all known methods fail we need to develop new ones. The next chapter is dedicated to the development of new techniques to attack the TDRW problem.

3.1.2 STATE OF THE ART (MDRW)

The history of RW dates back to Laplace. One dimensional RW has been throughly explored [SPIT64, KEMP61, FELL63]. However the multi-dimensional RW (MDRW) remaina mostly terra-incognita to this day.

The first application of RW models, of which the author is aware, appears in Lord J.W.S Rayleigh's work [RAYL80, RAYLE87]. Rayleigh applied RW models to the study of ensembles of waves, all possessing the same frequency and amplitude but having randomly distributed phases. In the course of his investigation, Rayleigh developed the diffusion approximation which he used to derive a multi-dimensional central limit theorem. The name "Random Walk" was coined only two decades later by K. Pearson [PEAR05] who initiated the study of the one-dimensional RW.

The next stage in the development of MDRW was achieved by G. Polya [POLY21]. Polya solved the recurrence problem for unbounded MDRW. His celebrated result is that the unbounded, symmetric nearest- neighbor RW is recurrent iff the dimension of the state space is smaller than three. Polya's work provided the impetus for research into recurrence of RW on some general state spaces. The research into unbounded MDRW culminated in the work of Dvoretzky-Erdos [DVOR51] and J.F Kingman [KING63].

The study of MDRW in the presence of boundaries, started with the work of Courant-Freidrich-Lewy [COUR28]. The authors were interested in discretizations of boundary value problems for partial differential equations. In their celebrated paper they point out the similarity of some boundary value problems of potential theory and heat flow to the problem of mean time to

absorption of MDRW in the presence of absorbing boundaries. The actual application of the idea was carried out by W.H. McCrea and F.J.W. Whipple [McCR36, McCR40] a few years later. V.D. Barnett [BARN63, BARN65] expanded this work connecting the potential-theoretic approach to a generalized form of Wald's identity.

The study of absorbing boundaries has been carried out by researchers from both fields: the probabilists who were able to apply methods of potential theory to attack problems of first hitting time; the numerical analysts who were able to utilize the analogy to apply monte-carlo methods to boundary-value problems of partial differential operators.

The solution of of the problem of absorbing boundaries yields a Green Function for the MDRW. It is possible to reduce any other boundary conditions into a system of singular integral equations with kernel given by the Green Function [SPIT64, KEIL65]. This relation between bounded MDRW and singular integral equations is essentially the underlying mechanism of our solution to the TDRW. However, we shall proceed to solve the problem directly and will not separate it into a problem of computing the Green Function and solving singular integral equations w.r.t. this kernel.

The problem of recurrence of MDRW in the presence of boundaries was treated by J.F Kingman [KING63]. Kingman applied Foster's stability criteria [FOST53] to generate simple, geometric, sufficient conditions for stability. Foster's criteria are expressed in terms of Liapunov type functions. Kingman's method is in essence a generalization of well known results about the stability of differential equations with boundary conditions [LEFS58]. We shall draw

upon Kingman's results in the sequel. Kingman's work was the first and, as far as this author could find, the last result in this direction.

The behavior of some special MDRW on infinite or periodic lattices has been explored and applied by physicists to problems of statistical mechanics of solids [MONT64, MONT73]. The results have been extended to handle some simple boundary behavior such as that arising in a defected lattice.

There have been few attempts to extend fluctuation theory to many dimensions, of which the most important development is probably the results of C. Hobby and R. Pyke [HOBB63]. Unfortunately, the combinatorial approach poses insurmountable difficulties when one tries to approach the MDRW through extensions of Ballot theorems to many dimensions. Similar difficulties are encountered when one tries to extend Renewal theory to many dimensions. The results obtained are very limited (for example see [HUNT74]).

Finally, the behavior of a MDRW in the presence of general boundaries is almost completely open. The first attack on a bounded MDRW is due to J.F. Kingman [KING61]. Kingman solved the problem of two queues in parallel, where an arriving customer joins the shortest queue. Kingman's method of attack is tailored to the two queues problem and does not lend itself in any simple manner to generalizations. Kingman's promise to generalize the method to attack general two dimensional problems (at the end of his paper) has, so far, never been fulfilled.. It took researchers more than fifteen years to understand Kingman's method [MCKE77].

We discovered Kingman's paper at the spring of 1976. Our work is in some

limited sense, a generalization of Kingman's solution process. We noticed a similarity between Kingman's solution and the Wiener-Hopf technique. At the end of 1976 we had already generalized the Wiener-Hopf technique to solve the general nearest neighbor positive TDRW, where diagonal movements are excluded. The solution [YEMI77] is carried through a set of transformations of the problem to a functional equation over a torus and may be explicitly written in terms of Jacobian elliptic functions. Since then we were able to derive the solution to the more general TDRW. The complete solution is presented in the next chapter from both a geometric and algebraic point of view.

Recently the problem of interacting queueing process has attracted a few researchers. This interest have been spurred by both practical needs and the rediscovery of Kingman's work. At the end of last summer Guy Fayolle, from IRIA, brought to our attention his work [FAYO77] on the problem of TDRW with no diagonal movements. Fayolle has successfully reduced the problem to a Riemann-Hilbert problem , then applied well known integral representations of the solution to the later. A somewhat simpler reduction process will be presented here for the sake of completeness. However, the general TDRW problem does not reduce to a simple Riemann-Hilbert problem but to a Riemann-Hilbert problem with a shift. This is proved in the next chapter. The solution of Riemann-Hilbert problems with a shift requires further sophistication. Therefore rather than pursuing a reduction process We prefer a method of direct attack which exposes the full solution directly. The ultimate goal should be to develop a solution algorithm which is completely algebraic. Fayolle also brought to our attention the paper by the Russian mathematician V. A. Malyshev [MALY72]. Malyshev's work is carried in full abstraction and

seems to be far from the concrete level at which we wish the solutions to be given. It provides a theoretical complementary study which together with Kingman's work, Mckean's, Fayolle's and ours forms a complete solution of the TDRW.

To summarize even simple MDRW poses extremely difficult problems for solution. We do not know how to handle MDRW in three or more dimensions. In two dimensions we do not know how to solve non nearest neighbor walk (although our solution process could be generalized to handle such problems). We do not know how to handle different geometries of the boundaries. The solutions that we shall present tend to become very complex. The problem of obtaining closed form solution to the general problem is most probably hopeless. However, can we get some good working approximate methods of solution? The works which were described above present different aspects and views of the problem (modulo some natural overlaps). It seems that we currently possess enough understanding to approach the more general MDRW problem.

3.1.3 THE EVOLUTION OF TWO DIMENSIONAL RANDOM WALKS

In the sequel we use the name TDRW and notation Q^t to denote the most general nearest-neighbor two dimensional RW, restricted to the closed positive quadrant. The general transition probabilities are depicted in Figure 3.1-3. Let $\pi^t(Q)$ be the probability distribution of the queueing process Q^t.

We define the following transforms:

$$A^{11}(z,w) \triangleq \begin{bmatrix} w & 1 & 1/w \end{bmatrix} \begin{bmatrix} \alpha_6 & \alpha_5 & \alpha_4 \\ \alpha_7 & \alpha_0-1 & \alpha_3 \\ \alpha_8 & \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} 1/z \\ 1 \\ z \end{bmatrix}$$

$$A^{10}(z,w) \triangleq \begin{bmatrix} w & 1 & 1/w \end{bmatrix} \begin{bmatrix} \alpha'_6 & \alpha'_5 & \alpha'_4 \\ \alpha'_7 & \alpha'_0-1 & \alpha'_3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/z \\ 1 \\ z \end{bmatrix} \qquad (3.1\text{-}1)$$

$$A^{01}(z,w) \triangleq \begin{bmatrix} w & 1 & 1/w \end{bmatrix} \begin{bmatrix} 0 & \alpha_5 & \alpha_4 \\ 0 & \alpha_0-1 & \alpha_3 \\ 0 & \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} 1/z \\ 1 \\ z \end{bmatrix}$$

$$A^{00}(z,w) \triangleq \begin{bmatrix} w & 1 & 1/w \end{bmatrix} \begin{bmatrix} 0 & \alpha_5 & \alpha_4 \\ 0 & \alpha_0-1 & \alpha_3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/z \\ 1 \\ z \end{bmatrix}$$

It will also be useful to employ a graphical notation for the above transforms which emphasizes the geometry of the associated motions. We shall use the following notation for the above transforms:

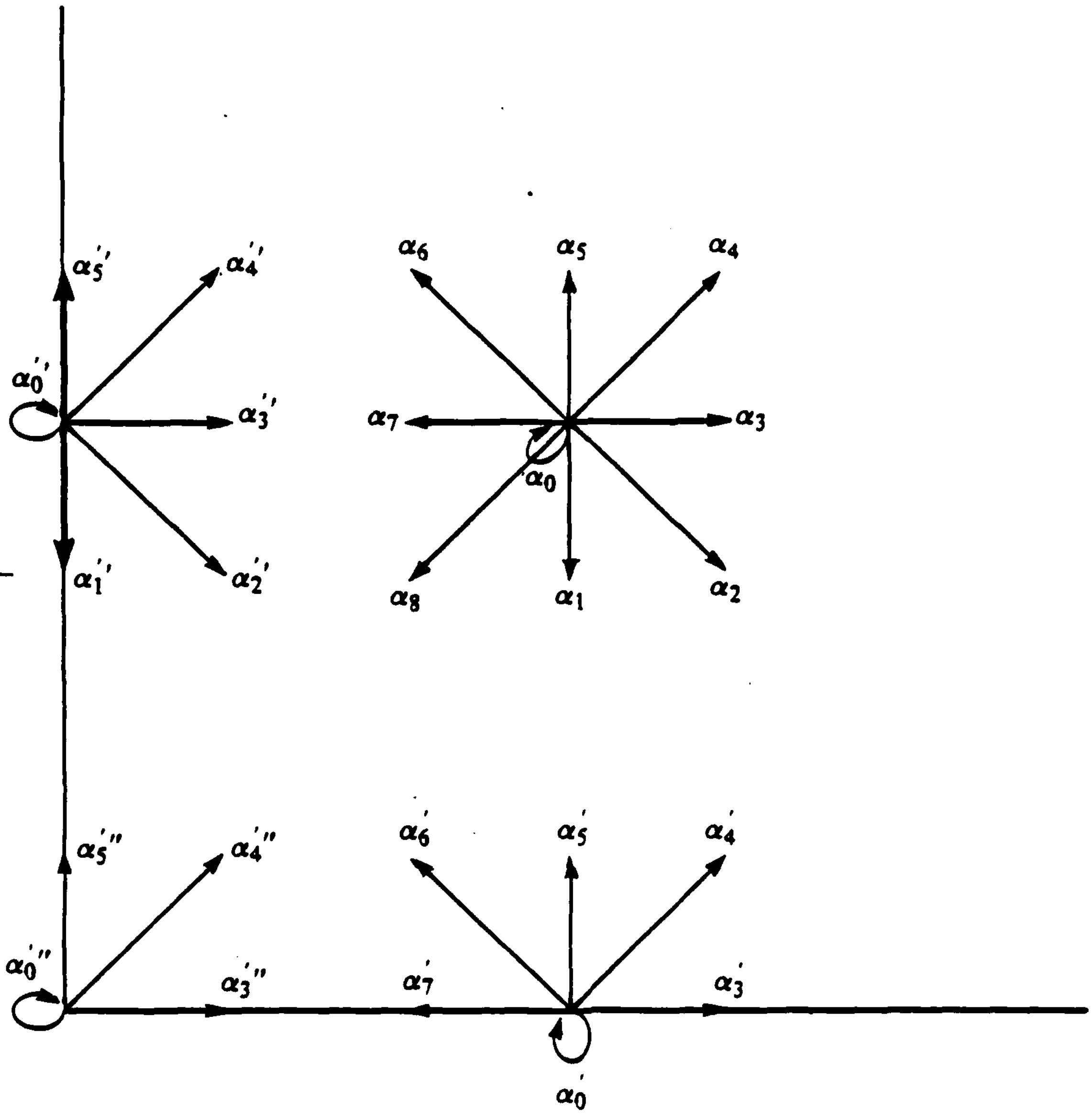
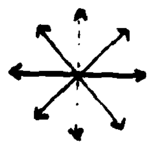
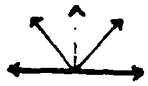
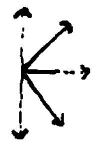


Figure 3.1-3: Transition probabilities for the general TDRW

$A^{11}(z,w) \triangleq$ 

$A^{10}(z,w) \triangleq$ 

$A^{01}(z,w) \triangleq$ 

$A^{00}(z,w) \triangleq$ 

In terms of these transforms, the evolution of the RW in time is completely described by:

$$\hat{G}^{t+1}(z,w) = \hat{G}^{t}(z,w) + A^{11}(z,w)\,G^{t}(z,w) + A^{10}(z,w)\,G^{t}_{10}(z) + A^{01}(z,w)\,G^{t}_{01}(w) + A^{00}(z,w)\,G^{t}_{00} \tag{3.1-2}$$

Where:

$$\begin{aligned}
G^{t}(z,w) &\triangleq \sum_{Q_2\geq 1}\sum_{Q_1\geq 1} \pi^{t}(Q_1,Q_2)\, z^{Q_1} w^{Q_2}\\
G^{t}_{10}(z) &\triangleq \sum_{Q_1\geq 1} \pi^{t}(Q_1,0)\, z^{Q_1}\\
G^{t}_{01}(w) &\triangleq \sum_{Q_2\geq 1} \pi^{t}(0,Q_2)\, w^{Q_2}\\
G^{t}_{00} &\triangleq \pi^{t}(0,0)
\end{aligned} \tag{3.1-3}$$

And

$$\hat{G}^t(z,w) \triangleq G^t(z,w) + G_{10}^t(z) + G_{01}^t(w) + G_{00}^t =$$

$$= \sum_{Q_1 \geq 0} \sum_{Q_2 \geq 0} \pi^t(Q_1, Q_2)\, z^{Q_1} w^{Q_2} =$$

$$= E[z^{Q^t_1} w^{Q^t_2}]$$

If the RW is stable (ergodic) [COX65] then, as t grows to infinity, $\pi^t(Q)$ converges to $\pi(Q)$; $\pi(Q)$ being the steady state distribution.

Let us define the following limiting transforms:

(3.1-4)

$$G^{11}(z,w) \triangleq \lim_{t \to \infty} G^t(z,w)$$

$$G^{10}(z,w) \triangleq \lim_{t \to \infty} G_{10}^t(z,w)$$

$$G^{01}(z,w) \triangleq \lim_{t \to \infty} G_{01}^t(z,w)$$

$$G^{00} \triangleq \lim_{t \to \infty} G_{00}^t$$

The steady state behavior of the RW is completely described by the following functional equation:

(3.1-5)

$$0 = A^{11}(z,w)\, G^{11}(z,w) + A^{10}(z,w)\, G^{10}(z) + A^{01}(z,w)\, G^{01}(w) + A^{00}(z,w)\, G^{00}$$

This equation in the four unknowns $G^{11}(z,w)$, $G^{10}(z)$, $G^{01}(w)$ and G^{00}, contains all the information that we have about the steady state behavior of the RW.

That is, the RW is stable iff equation 3.1-5 possesses a unique (up to a multiplicative factor) solution, G^{11}, G^{10}, G^{01} and G^{00}, such that

(3.1-6) $\quad G^{11}(z,w)$ is analytic in the polydisk $D_z(1)\times D_w(1)$;

$G^{10}(z)$ and $G^{01}(w)$ are analytic in $D_z(1)$ and $D_w(1)$, respectively.

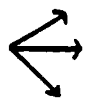
Here $D_z(a) \triangleq \{z: |z| < a\}$ and $D_w(a)$ is defined similarly.

The determination of the solution of 3.1-5 which satisfies the analyticity conditions 3.1-6, is our major problem.

The main idea which we pursue is to consider the algebraic curve:

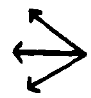
$$0 = A^{11}(z,w) = zp(w) + q(w) + r(w)/z = \tag{3.1-7}$$

$$= ws(z) + t(z) + v(z)/w$$

where $\quad p(w) \triangleq \begin{bmatrix} w & 1 & 1/w \end{bmatrix} \begin{bmatrix} \alpha_4 \\ \alpha_3 \\ \alpha_2 \end{bmatrix} =$ 

$$q(w) \triangleq \begin{bmatrix} w & 1 & 1/w \end{bmatrix} \begin{bmatrix} \alpha_5 \\ \alpha_0 - 1 \\ \alpha_1 \end{bmatrix} = \tag{3.1-8}$$

$$r(w) \triangleq \begin{bmatrix} w & 1 & 1/w \end{bmatrix} \begin{bmatrix} \alpha_6 \\ \alpha_7 \\ \alpha_8 \end{bmatrix} =$$



and $s(z)$, $t(z)$, $v(z)$ are defined similarly in terms of z.

We call the curve defined by equation 3.1-7 the *characteristic curve* of the RW.

On this curve the steady state equation reduces to:

$$\text{(3.1-9)} \qquad 0 = A^{10}(z,w)G^{10}(z) + A^{01}(z,w)G^{01}(w) + A^{00}(z,w)G^{00}$$

We call this functional equation: the *boundary equation*. The idea which we pursue is to solve the boundary equation first, then use the results to obtain a solution for the steady state equation 3.1-5.

Sometimes, it will be more convenient to consider a different form of expressing the steady state equation 3.1-5. We shall employ one unknown function only

$$G(z,w) \triangleq G^{11}(z,w) + G^{10}(z) + G^{10}(w) + G^{00}$$

In terms of this unknown function, the steady state equation assumes the form

$$\text{(3.1-10)} \qquad A^{11}G(z,w) = \hat{A}^{10}G(z,0) + \hat{A}^{01}G(0,w) + \hat{A}^{00}G(0,0)$$

Here the coefficients are readily computed to be:

$$\hat{A}^{10}(z,w) \triangleq A^{11}(z,w) - A^{10}(z,w)$$

$$\hat{A}^{01}(z,w) \triangleq A^{11}(z,w) - A^{01}(z,w)$$

$$\hat{A}^{00}(z,w) \triangleq A^{10}(z,w) + A^{01}(z,w) - A^{10}(z,w) - A^{00}(z,w)$$

We shall return to these transform relations in the fourth section of this chapter, where we solve the "maximum interference" model of the 2-buffered queues, and more heavily in the next chapter, where we solve equation 3.1-5 for a number of cases of interest.

3.1.4 SIMULATION RESULTS FOR THE FOUR MODELS

The four TDRWs corresponding to the four models have all been simulated for the case of two PRUs which are completely symmetric, i.e have the same arrival rate and transmission probabilities. Figure 3.1-4 depicts the typical delay-throughput relations for a PRU in the four systems. Figure 3.1-5 illustrates the relation between input rate and throughput for the four models. The transmission probabilities chosen for simulation are 0.5 (stability considerations show that the input rate to each queue should not exceed 0.25).

The steady state distributions of the number of queued packets are excellently approximated by geometric distributions. More precisely, if $\pi(n)$ is the steady state distribution of the number in queue, then we consider the function $\log\pi(n)$. If $\pi(n)$ were geometric with parameter ρ, i.e., $\pi(n)=(1-\rho)\rho^{n}$, then $\log\pi(n)=\log(1-\rho)+n\log\rho$; thus, $\log\pi(n)$ would have been linear in n. In fact, the logarithm of the simulated $\pi(n)$ turns out to be excellently approximated by a straight line. Moreover, the values of the respective parameter ρ, as computed from the direction of the line ($\log\rho$) and its value at $n=0$ ($\log(1-\rho)$) seem to be in excellent agreement. We call the parameter ρ, so computed, the *utilization* parameter of the queue (for obvious reasons). The "utilization" parameter for the geometric fit of the steady state distribution is depicted in Figure 3.1-6 as a function of the input rate. The accuracy of the geometric fit motivated our attempts to develop a "one pole" (i.e., geometric) approximation to the transform of the distribution.

A surprising effect is the sensitivity of the system to a change in the hearing relations. The increase in interference from the first system to the fourth

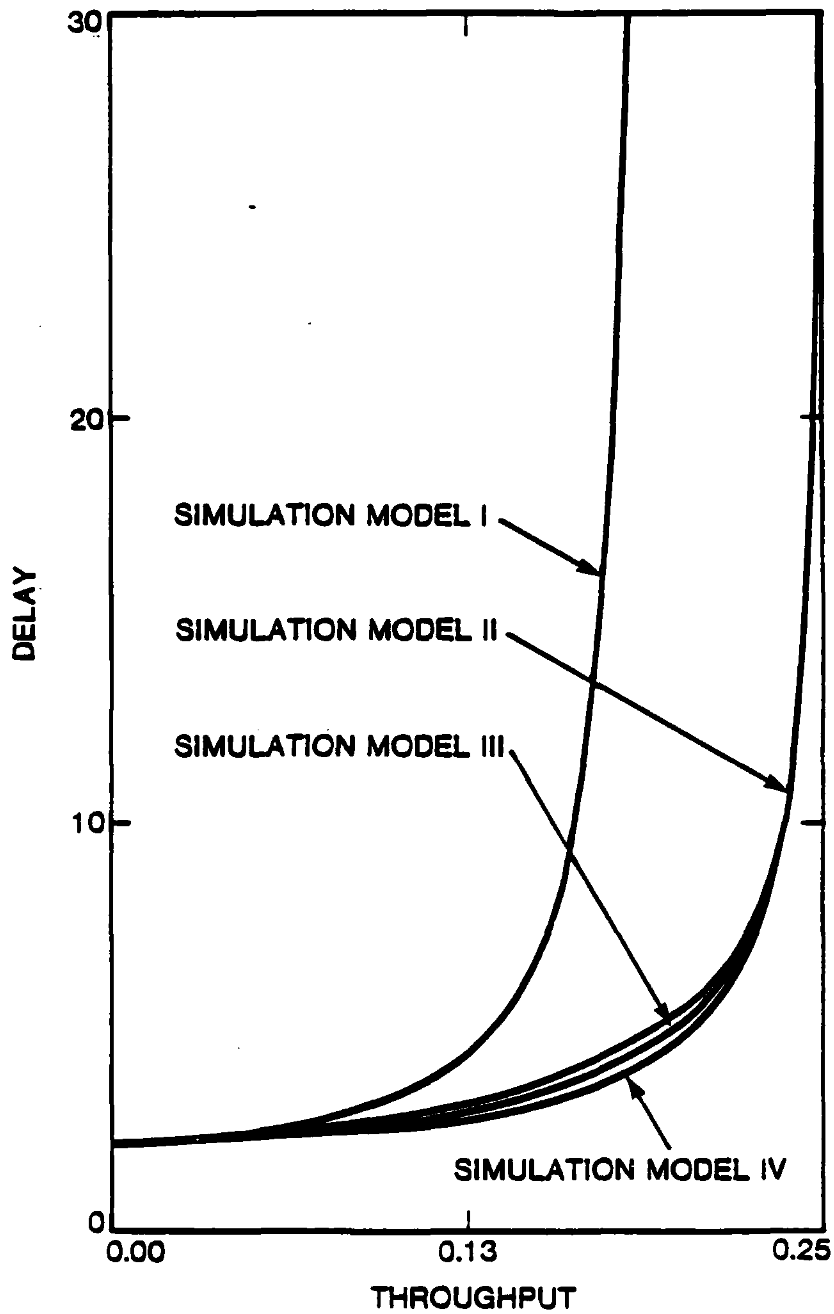


Figure 3.1-4: Throughput-Delay for the four models

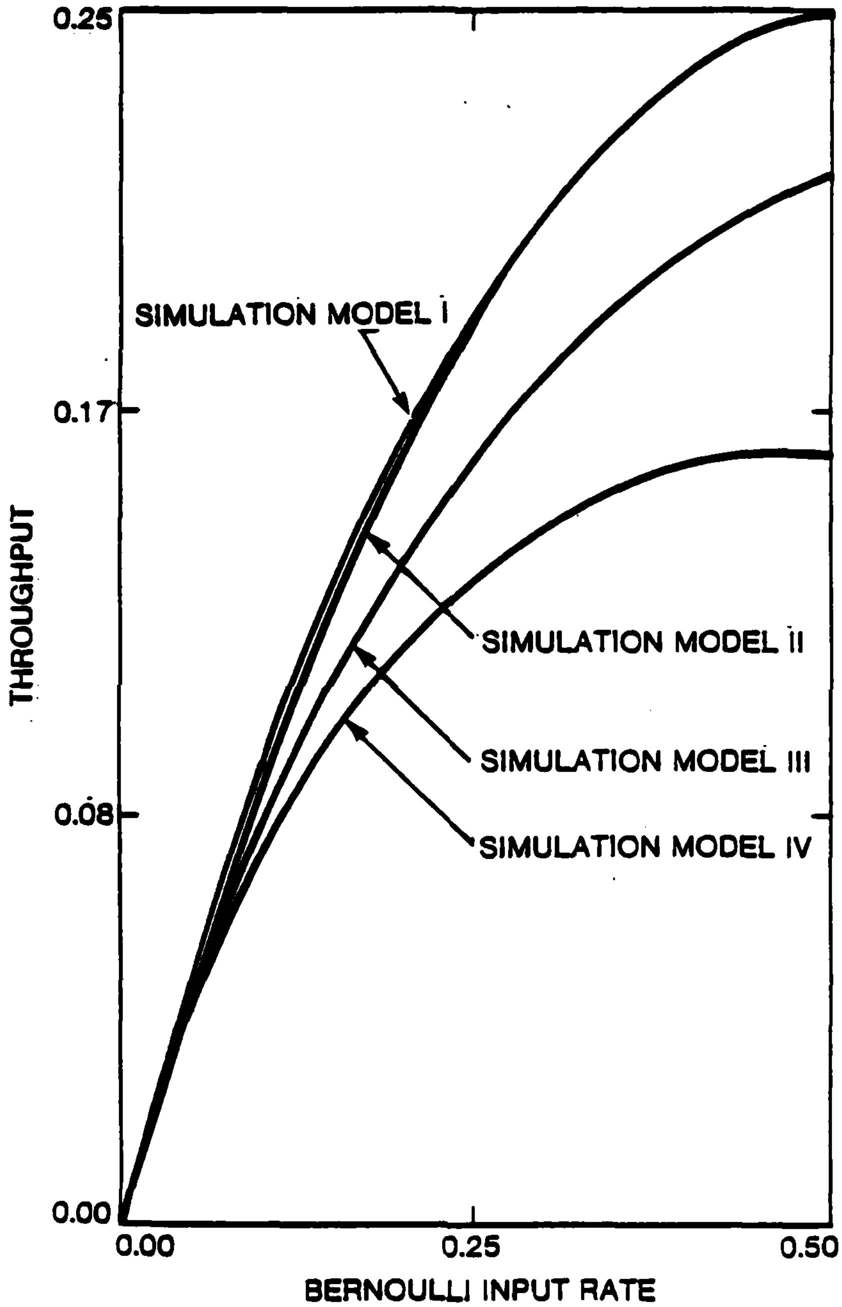


Figure 3.1-5: Throughput-Offered load for the four models

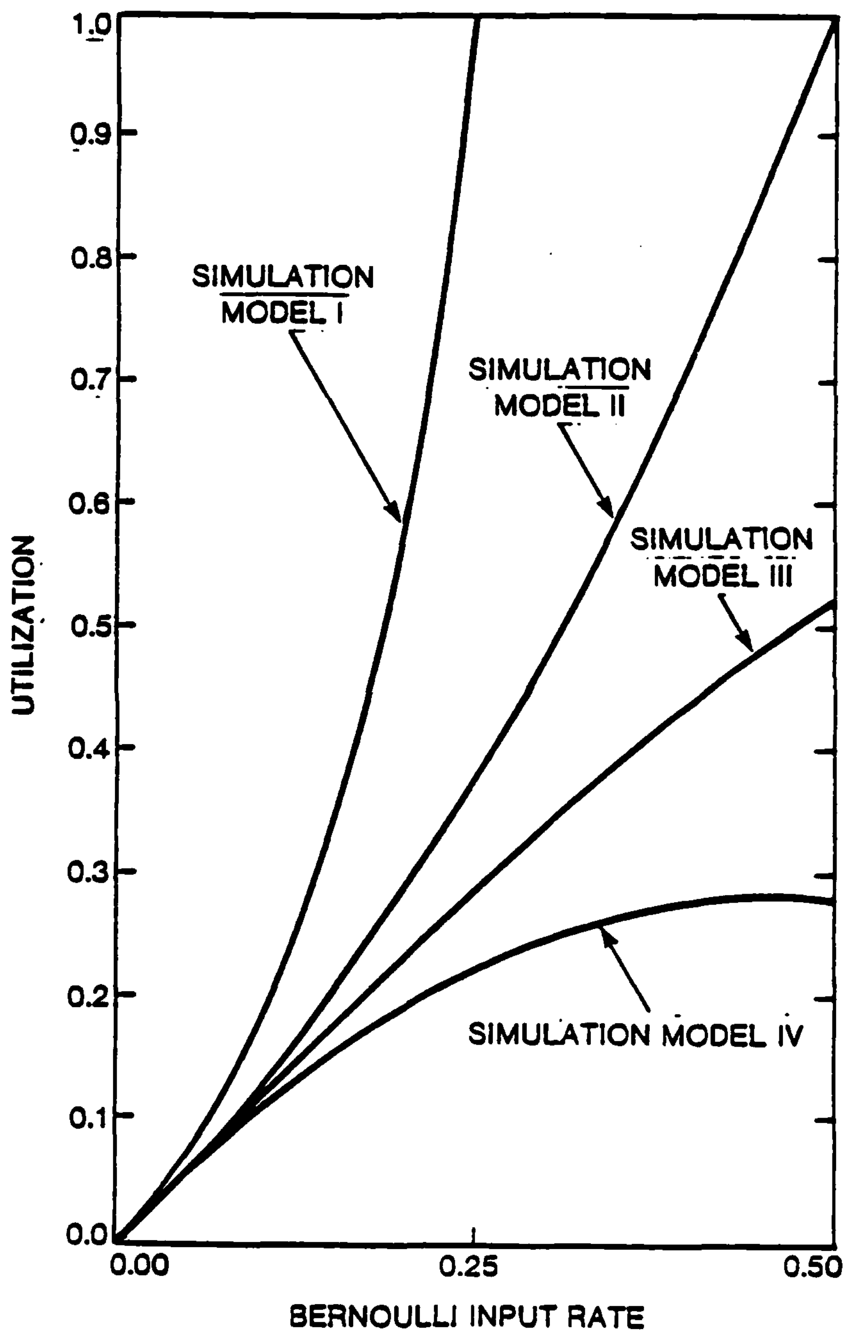


Figure 3.1-6: "Utilization" parameter for the four models

improves the delay-throughput performance (paradoxically). The passage from the first model to the second model improves the performance significantly in spite of the fact that the two models are almost the same. However, the passage from the second model to the third and fourth hardly affects the delay-throughput curves. Since the fourth model may be solved in terms of simple formulae (see section 3.4.3), it can serve as an excellent physical approximation to the second and third model.

The process may be generalized to any number of PRUs. That is, we may use maximum interference models as approximations of other interference structures, knowing that the approximation is excellent beyond certain threshold of interference. It is left for a future research to characterize this threshold behavior.

3.2 WHAT MAKES THE PROBLEM SO HARD ?

3.2.1 KINGMAN'S ALGEBRA OF QUEUES

In this section we consider our problem from an algebraic point of view. We employ J. F. Kingman's [KING63] unified view of queueing theory. As we shall see, none of the methods that solve one dimensional queueing problems, may solve our problem. The immediate conclusion is that we should try approximations and/or develop a completely new set of tools to solve our problem.

First, let us briefly review Kingman's algebra of queues. Kingman's point of departure is the equation for the evolution in time of the waiting time in a G/G/1 queueing process

(3.2-1) $\quad w_n = [w_{n-1} + u_n]^+$

Here, w_n is the waiting time of the n-th customer; u_n is the excess of service time for the n-th customer, over the arrival time of his successor. The function $[x]^+$ assumes the value x if x>0 and 0 otherwise.

The evolution equation for the probability distribution of the waiting time process is given, respectively, by

(3.2-2) $\quad g_n = \prod (g_n * f_n)$

Here g_n is the distribution of w_n; f_n is the distribution of u_n ; "*" is the convolution operation and $\prod$ is a "sweeping" operator which takes all the mass of a given measure which is concentrated on the negative numbers and "sweeps" it to (i.e., places it at) the origin.

Kingman shows that the operator $\prod$, when considered over the algebra M of all finite signed measures on E (E is a one dimensional Euclidian space), is a projection with an interesting closure relation to the convolution operation. Namely, the range and the null space of the projection are closed w.r.t. convolution (thus forming a decomposition of M into a direct sum of two sub-algebras). Projections possessing this property are called Wendel projections. The methods of solving one dimensional queueing problems are demonstrated to follow from this particular algebraic structure of the operator $\prod$. Indeed, all major schemes for solving one dimensional queues are shown to be but different realizations of the same algebraic factorization process applied to Wendel projections.

Towards the end of the monograph ([KING63] page 49) Kingman examines the question of characterization of Wendel Projection operators for general Euclidian spaces. In particular, if the function $[\ ]^+$ is replaced by any function $\phi: \mathbb{R}^k \text{----}> \mathbb{R}^k$, and if $\prod$ is the corresponding operator on the algebra of finite signed borel measures on $\mathbb{R}^k$, then a necessary and sufficient condition that we can apply the factorization process to the operator $\prod$, is that the function ϕ satisfies the condition

(3.2-3) For all $x,y \in \mathbb{R}^k$, the two pairs:

$$\phi(x+y),\ \phi(x)+\phi(y) \text{ and } \phi(x+\phi(y)),\ \phi(\phi(x)+y) \quad \text{are equal}$$

It can be readily shown that G/G/k problems cannot be solved using any of the G/G/1 schemes because the associated projection operator is not a Wendel projection. Kingman summarizes his finding " *This would seem to imply that some radically new idea will be needed before one can hope for a general solution to G/G/k*".

3.2.2 PROJECTION OPERATORS ASSOCIATED WITH BOUNDED TDRW

Let us apply Kingman's approach to some typical, two dimensional, simple, positive RW. We reconsider the positive RW and boundary behavior which results from some simple "sweeping operator". Figure 3.2-1 represents the simplest sweeping operator, i.e., when the walk tries to cross the boundary towards the forbidden region, it is immediately projected onto the respective axis. The corresponding projection is given by:

$$(3.2\text{-}4) \qquad \phi_1 : \mathbb{R}^2 \dashrightarrow \mathbb{R}^2$$
$$\phi_1(x,y) \triangleq ([x]^+,[y]^+)$$

It is easy to verify that the function $\phi_1(x,y)$ is a projection on $\mathbb{R}^2$. The corresponding sweeping operator is a projection on the algebra of signed measures on $\mathbb{R}^2$. The range of the projector is the set of measures concentrated on the closed positive quadrant. The kernel of the projection consists of measures concentrated on the closure of the third (negative) quadrant, and whose total mass is zero (so that they are swept to a zero measure). It is easy to check that the range and the kernel are closed w.r.t. convolution. Therefore the sweeping projector is a Wendel projection. The TDRW should be soluble in terms of classical methods. Indeed, in this case the one dimensional projections on the x, y axes, perform independent, one dimensional TDRW.

Unfortunately, the simple model above is an exception. If we modify the boundary behavior even slightly, the resulting walk can no longer be solved using classical methods. For instance, let us consider the boundary behavior of Figure 3.2-2. An attempt to move into the forbidden region, results in no move. The corresponding projection is given by:

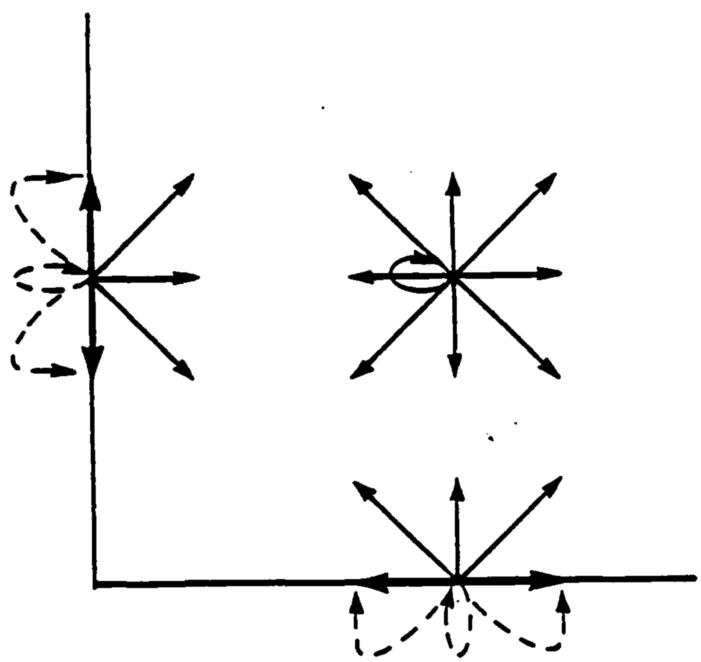


Figure 3.2-1: The "projected" TDRW

$$(3.2\text{-}5)\qquad \phi_2(x,y) \triangleq \begin{cases} (x,y) & \text{if } x\geq 0,\ y\geq 0 \\ (0,0) & \text{otherwise} \end{cases}$$

We show that the corresponding sweeping projector is not a Wendel projection. The range of the sweeping operator consists of measures supported on the closure of the positive quadrant. The kernel of the sweeping operator consists of the space of measures whose support is in the closure of the other three quadrants, and whose total mass is zero. This last space is not closed w.r.t. convolution. Thus, the sweeping operation is not a Wendel projection. This will follow if we exhibit two measures in the kernel, whose convolution product is not swept to the zero measure. An easy example is furnished by:

$$\mu_1 \triangleq \begin{cases} 1 \text{ at } (2,-1) \\ -1 \text{ at } (4,-1) \end{cases}$$

and

$$\mu_2 \triangleq \begin{cases} 1 \text{ at } (-1,2) \\ -1 \text{ at } (-1,4) \end{cases}$$

It is easy to see that the convolution $\mu_1 * \mu_2$ is concentrated at the points (1,1), (3,3), (1,3) and (3,1); thus, it is "swept" unto itself and not the zero measure.

Similarly, one may check that many other simple types of boundary behavior result in sweeping operators which fail to be Wendel projections. Therefore, the respective TDRW cannot be solved using factorization methods such as those employed for one-dimensional queueing processes. The inevitable conclusion is that we should be prepared to develop new tools to solve the TDRW problem.

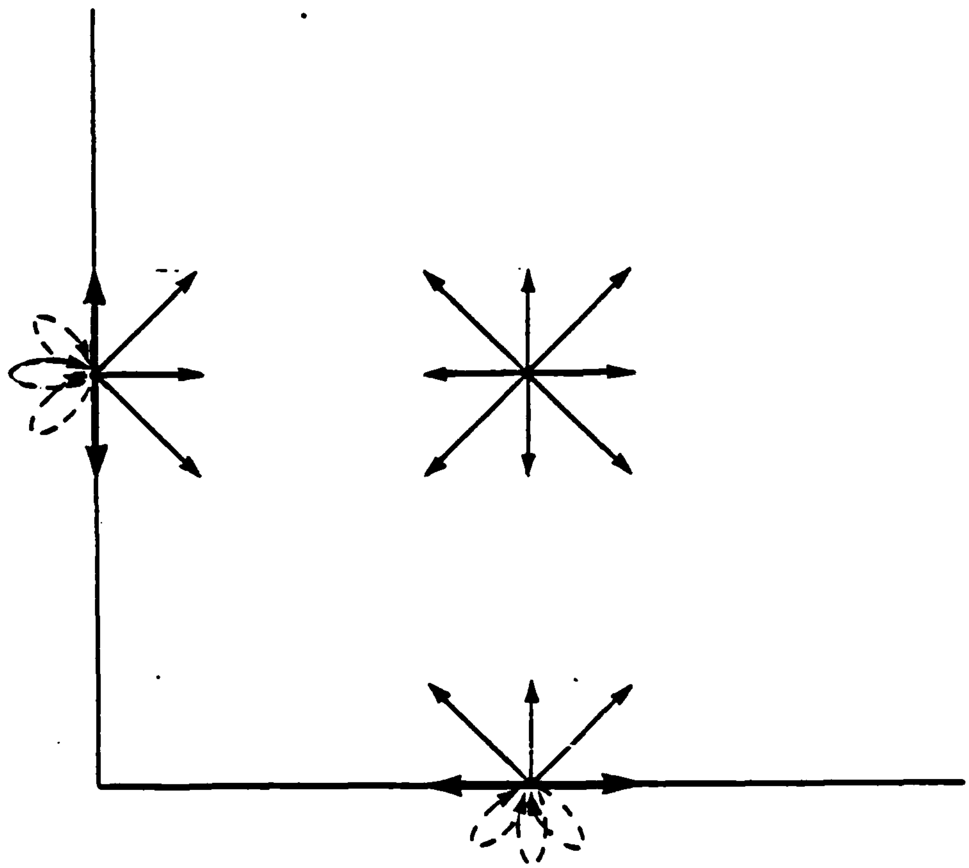


Figure 3.2-2: A "Sweeping" operator which is not a Wendel projection

However, before we venture into the difficult problem of an exact solution, let us try some simple approximations. This is the subject of the next section.

3.3 APPROXIMATE SOLUTIONS

3.3.1 HEAVY TRAFFIC APPROXIMATION

Our point of departure in developing approximate methods, is to simplify the relation between the boundary and interior behavior of the queueing RW. A reasonable simplification arises if we adjust the transitions at the boundaries so that the projections of the two dimensional walk on the two axes perform a one-dimensional RW. This is the Heavy Traffic approximation. A typical such walk is described in Figure 3.3-1. We shall call a TDRW whose projected movements are one-dimensional RW: *projectable walk*. The projected one-dimensional RWs are called : *marginal RW*. The solution for the marginal walks is trivial (i.e., the problem becomes identical to basic Queueing Theory).

Consider our first model of section 3.1.1. Under the Heavy Traffic assumption each PRU sees the other as a Bernoulli source of interfering noise. We ignore the details of the interaction of the two queues. The interaction is reduced to a constant interference. The queueing process at each PRU is the respective marginal RW associated with the TDRW.

Let us solve the general one-dimensional RW with transition structure as in Figure 3.3-2, then apply the results to the marginal RW of the heavy traffic approximation. The transformed steady state equation for the distribution of number in queue is given by:

$$(3.3\text{-}1) \qquad A^1(z)G^1(z) + A^\theta(z)G^\theta = 0$$

where

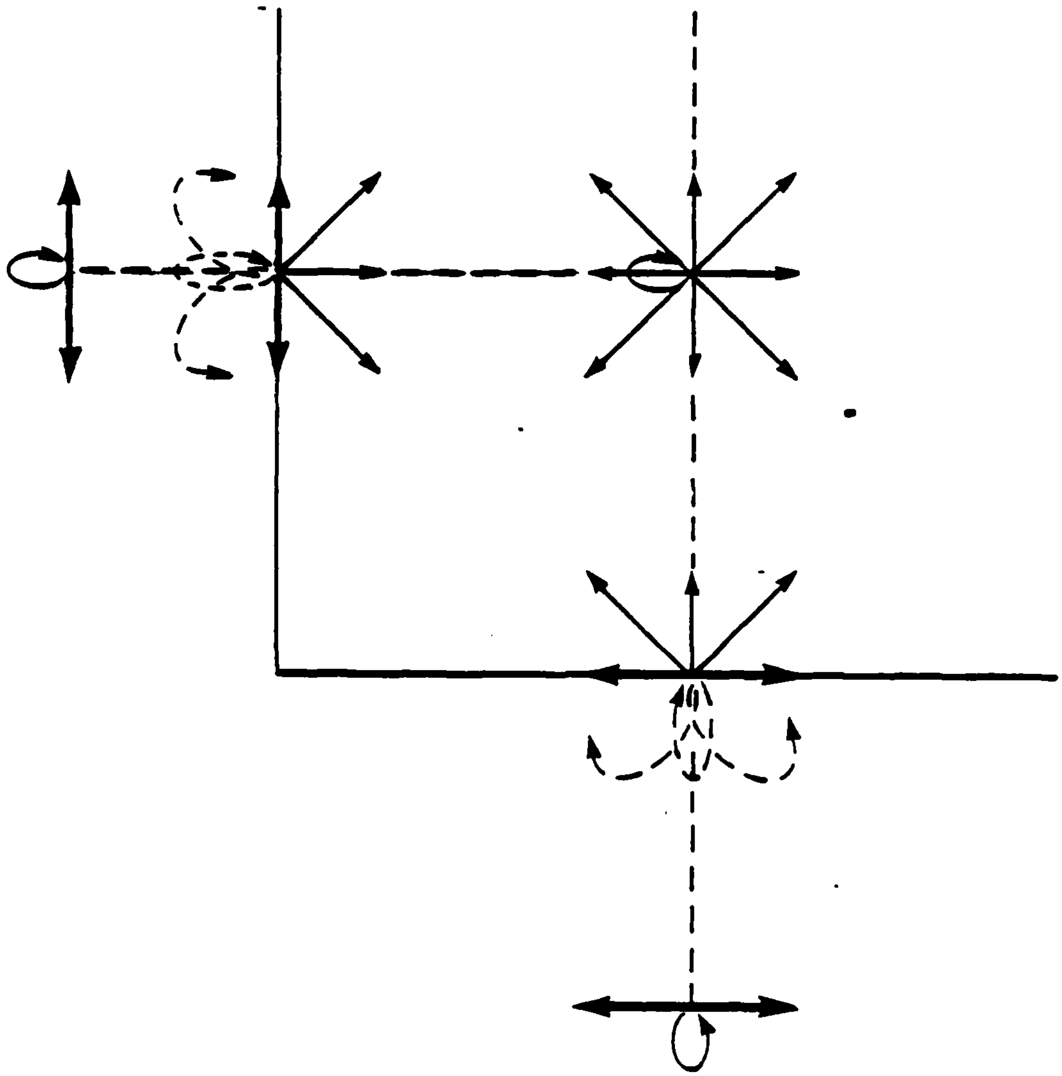


Figure 3.3-1: The projectable TDRW and its associated marginal RW

$$G^1(z) \triangleq \sum_{Q=1}^{\infty} \pi(Q)z^Q \qquad \text{and } G^0 \triangleq \pi(0)$$

($\pi(Q)$ is the steady state probability of the number in queue.)

$$A^1(z) \triangleq \alpha(1-z)\left(\frac{1}{\rho z} - 1\right), \qquad \rho \triangleq \alpha/\beta$$

and

$$A^0(z) \triangleq -(\alpha/\gamma)(1-z)$$

are the transforms of the respective one-step transition distributions.

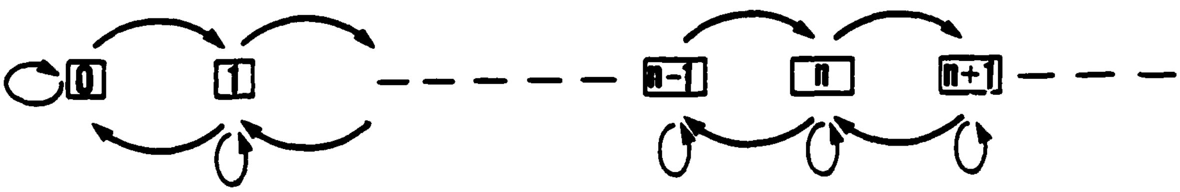


Figure 3.3-2: Transition diagram of a nearest-neighbor, positive RW

The solution of equation 3.3-1 is easily found to be

$$(3.3\text{-}2) \qquad G(z) \triangleq \quad G(z) \triangleq G^1(z) + G^0 = \frac{1/(1-\rho z) - \bar{\gamma}}{1/(1-\rho) - \bar{\gamma}}$$

The steady state distribution is geometric (modulo a perturbation of the probability of the state 0), with "utilization parameter":

$$(3.3\text{-}3) \qquad \rho = \alpha/\beta$$

The expected number in queue is given by

$$\bar{Q} = \frac{\rho}{(1-\rho)(\gamma + \bar{\gamma}\rho)} \tag{3.3-4}$$

The expected throughput (i.e., the average number of transitions to the left) is

$$\bar{S} = \alpha/(\gamma + \bar{\gamma}\rho) \tag{3.3-5}$$

Using Little's result we find the expected delay to be

$$T = \bar{Q}/S = 1/(1-\rho)\beta \tag{3.3-6}$$

(This last equation for the expected delay of a Bernoulli-input/Bernoulli-service/1 queueing process is, not surprisingly, similar to the corresponding M/M/1 formula.)

Now specialize the above formulae to the heavy traffic RW. The utilization parameters, the expected numbers of queued packets, the expected throughputs and the expected delays for the four models are given in table 3.3-1.

Let us consider the symmetric case with $\mu \triangleq \mu_1 = \mu_2 = 0.5$. Let $\lambda \triangleq \lambda_1 = \lambda_2$ be the arrival rate. The relation of the heavy-traffic and light-traffic (see next section) approximations to the results of simulation is described in Figures 3.3-3 to 3.3-14, below. The first collection of figures describes the respective utilization parameters as functions of the input rate. Both light and heavy traffic assumptions lead to a geometric steady state distributions for the number in each queue. The utilization parameter is the parameter of the respective geometric distribution. The utilization parameter of the steady state

distribution obtained from simulation is the parameter of the geometric fit to the actual distribution (the error in the approximation is extremely small).

	ρ_{heavy}	ρ_{light}
I	$\lambda(1-\mu\bar{\mu})/\bar{\lambda}\mu\bar{\mu}$	$\lambda\bar{\mu}/\bar{\lambda}_{\mu}$
II	$\lambda\ /\mu$	$\lambda\bar{\mu}/\mu$
III	$\lambda\bar{\mu}/\mu$	$\lambda\bar{\mu}/\mu$
IV	$\lambda\bar{\lambda}\bar{\mu}/\mu$	$\lambda\bar{\lambda}\bar{\mu}/\mu$

Table 3.3-1: Heavy and light traffic approximation for the four models

The second collection of figures depicts the relation between input rates and the actual throughputs delivered by each queue, as given by simulation and approximations. The third collection of figures compares the delay-throughput relations obtained from simulation and the heavy and low traffic approximations.

It may be seen that a simple approximation such as the heavy traffic approximation, provides a good fit to the actual behavior of the RW, only when the traffic is extremely heavy and/or the interference is heavy. However, when we wish to compare Slotted-Aloha with deterministic schemes the heavy traffic approximation is a bad solution. After all the use of Aloha has been considered as a service method for a light, bursty traffic only.

To conclude this section, we see that a better approximation is required if we wish to explore the interaction between the two queues.

3.3.2 LIGHT TRAFFIC APPROXIMATION

The failure of the heavy traffic approximation to provide an acceptable fit to simulation, leads to a search for a light traffic approximation. Here we make the assumption that the two queueing process are very lightly loaded. The effect of collisions is negligible and may be ignored (hopefully). We assume that no interaction between the queues arises.

The transition structure of the queueing RW at each PRU is similar to the general model of Figure 3.3-2. Let us specialize the analysis of the general one-dimensional model to the light traffic approximation. The steady state distributions of the number in queue are, again, geometric. The utilization parameters, expected numbers of queued packets, throughputs and delays are given in table 3.3-1.

The light traffic approximation has been compared to simulation. The results are depicted in Figures 3.3-3 to 3.3-14. It is seen that light-traffic approximation deviates from the actual performance even for very light traffic and low interference. Therefore it is unsuitable as an approximation method.

The comparison of the behavior predicted by our approximate results to the actual performance is disheartening. (The only surprisingly excellent approximation is given by the heavy-traffic approximation for the delay-throughput performance of strongly interfering models). We are lead to believe that the elimination of interaction between the boundaries and the interior, in our approximate models, should be improved in favor of more sophisticated models. After all it is this very interaction which renders the

ALOHA scheme useful. The whole idea of resource sharing by two bursty users is to trade the overhead and slower service when both queues are busy for the improved rate of service when they hit the boundary. Rather than eliminating this fruitful interaction we should try to solve for it.

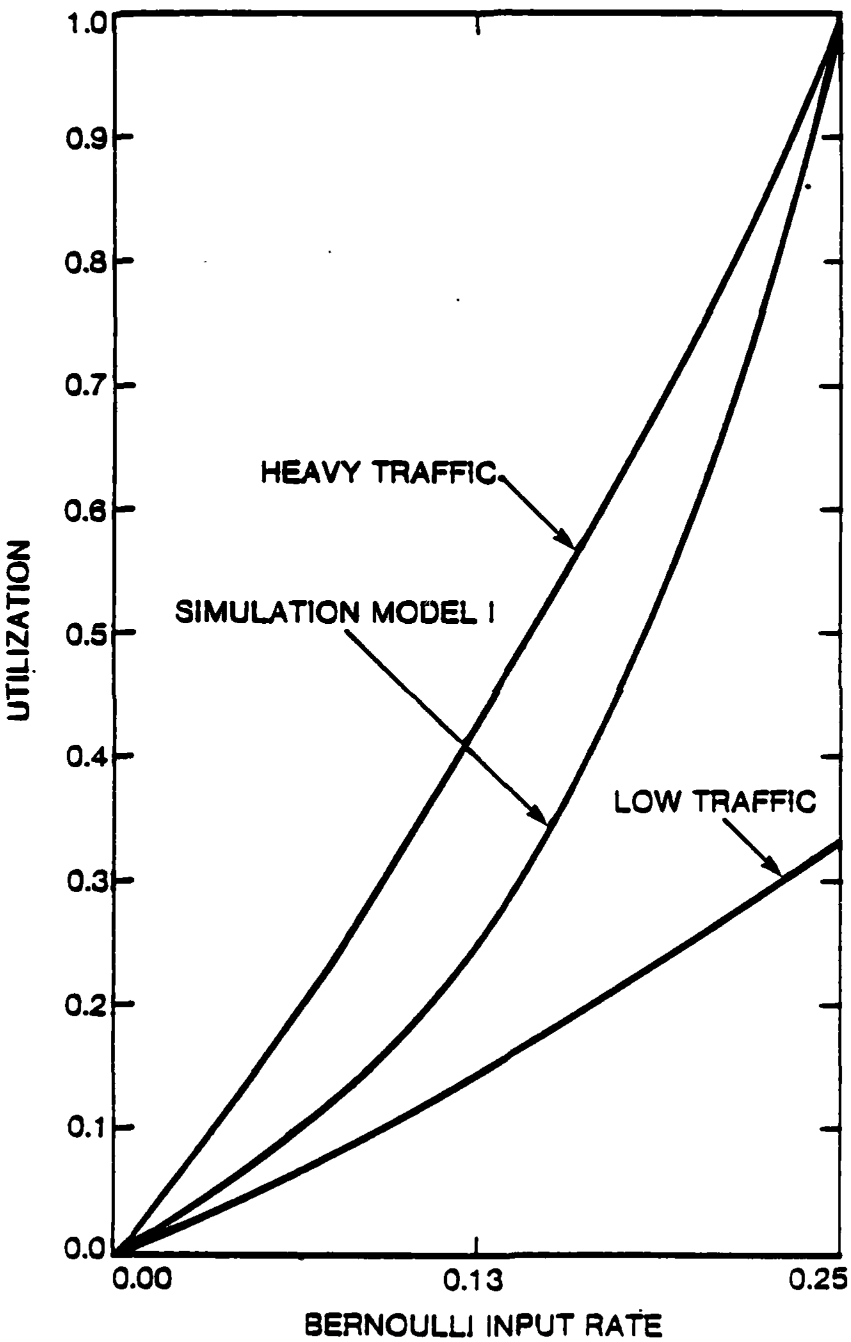


Figure 3.3-3: Heavy and Light traffic approximation of utilization; model I

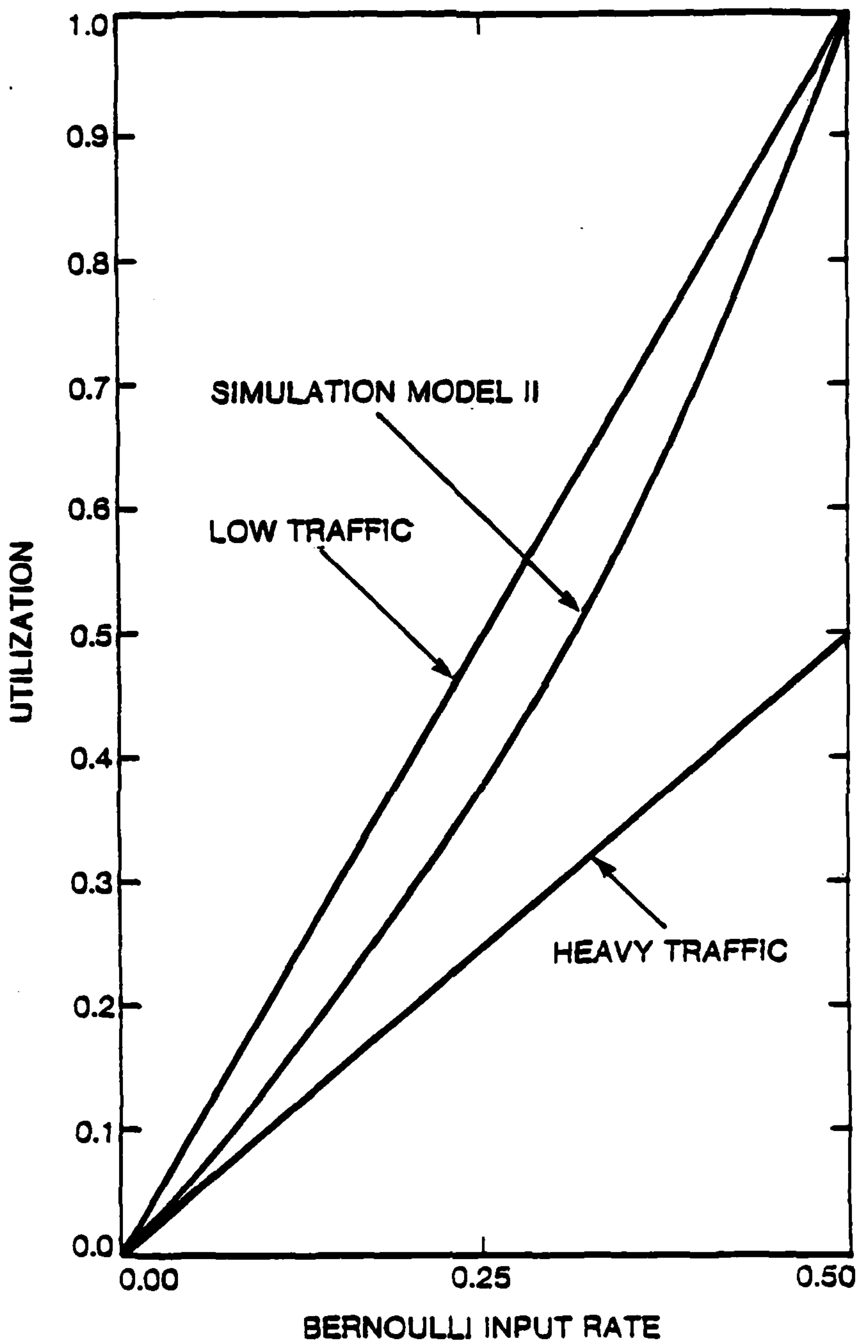


Figure 3.3-4: Heavy and Light traffic approximation of utilization; model II

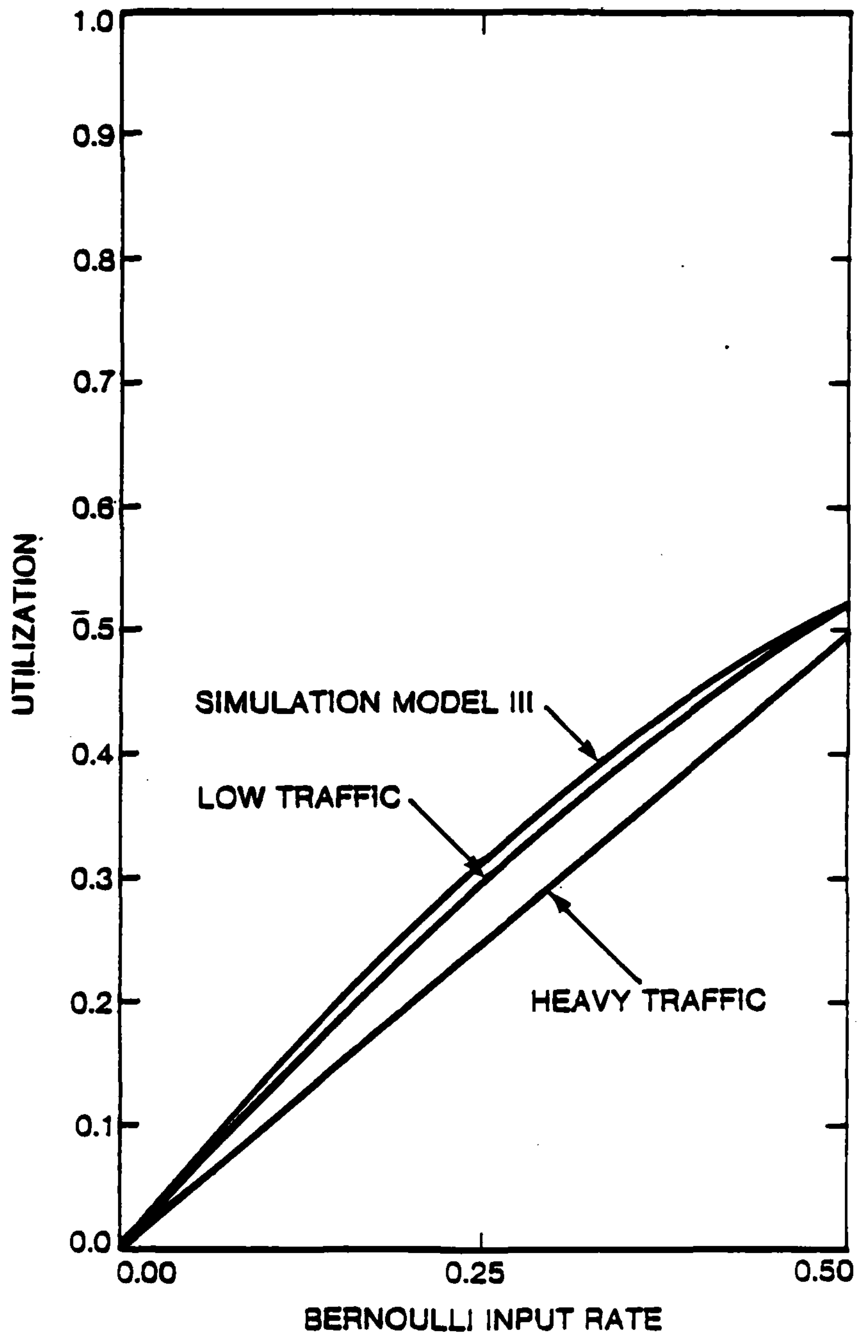


Figure 3.3-5: Heavy and Light traffic approximation of utilization; model III

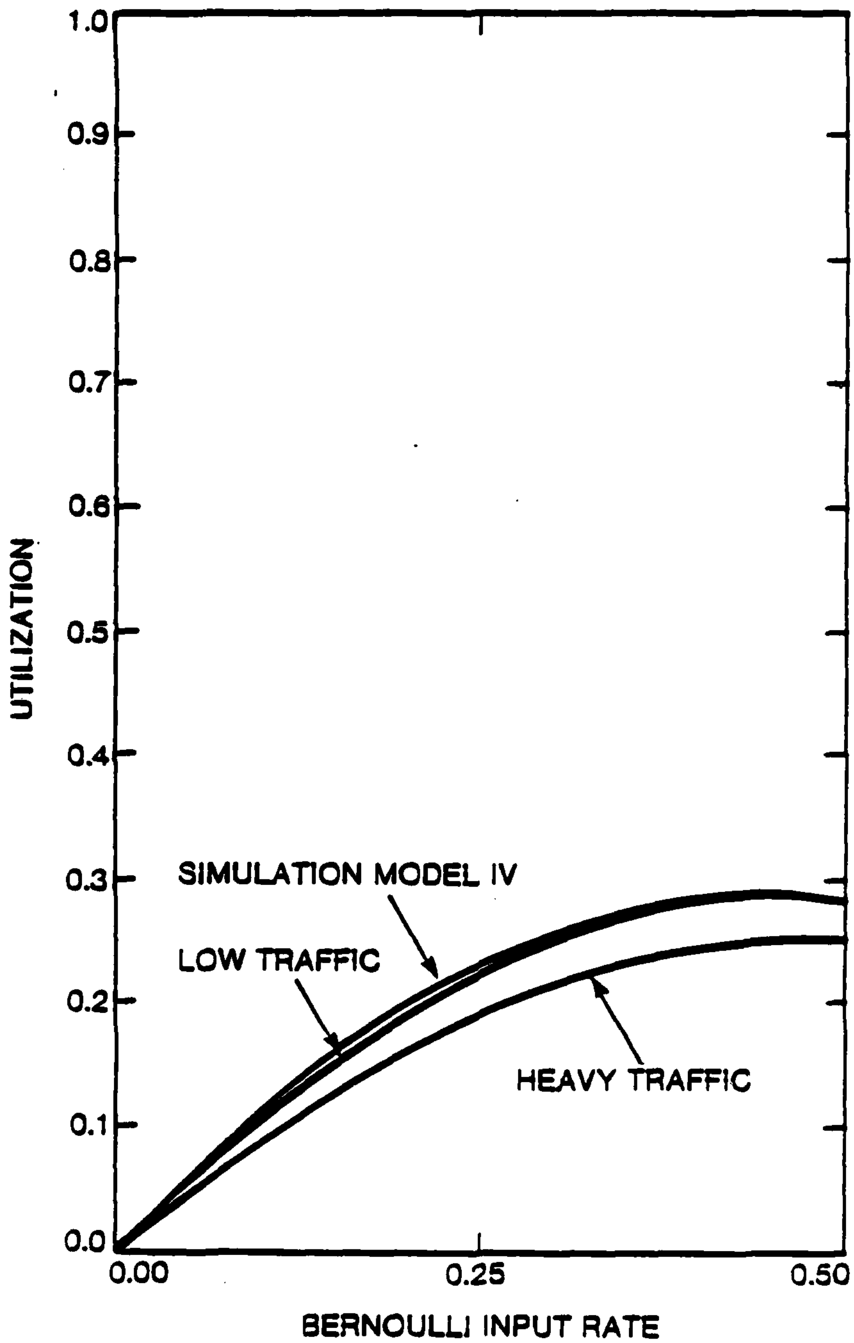


Figure 3.3-6: Heavy and Light approximation of utilization; model IV

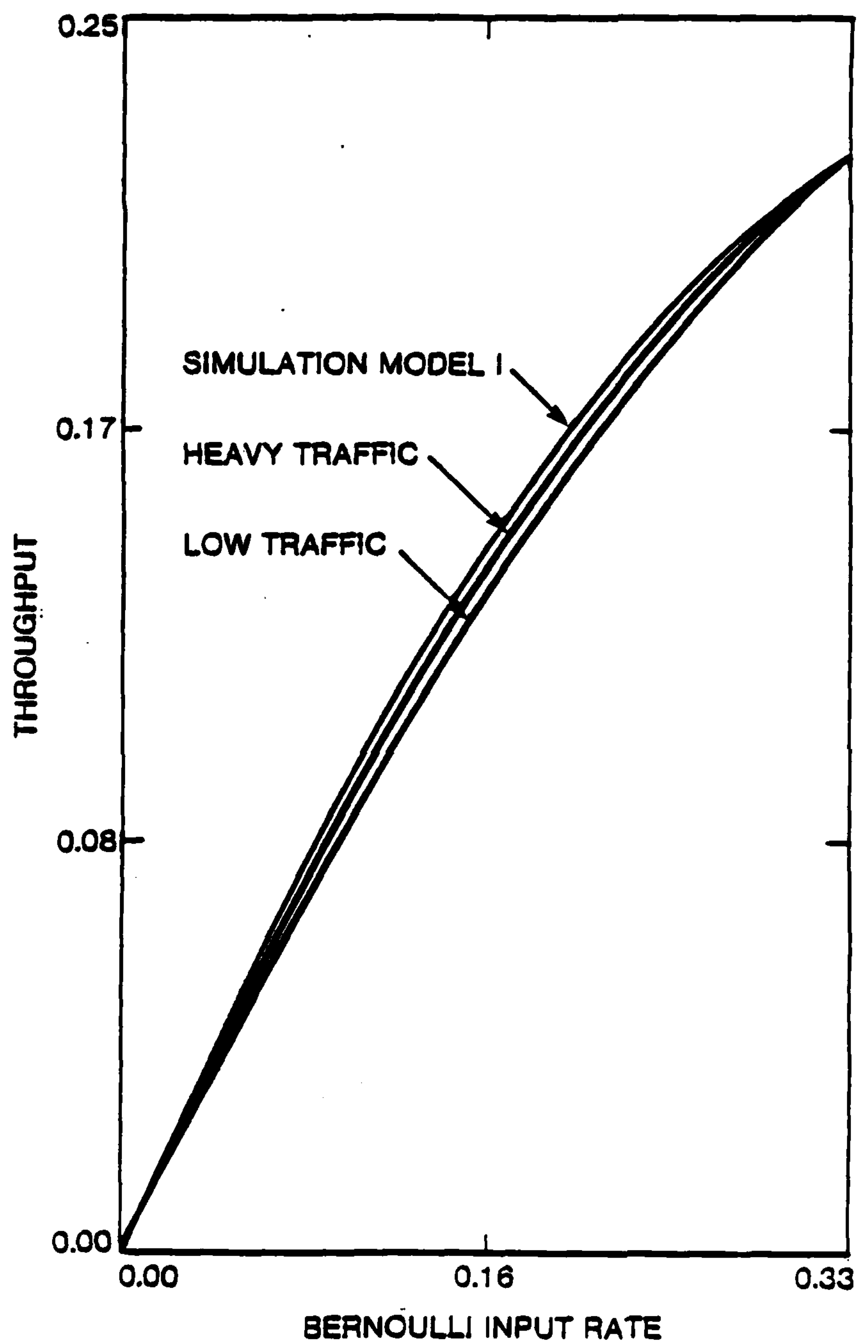


Figure 3.3-7: Heavy and Light traffic approximation of throughput-load; model I

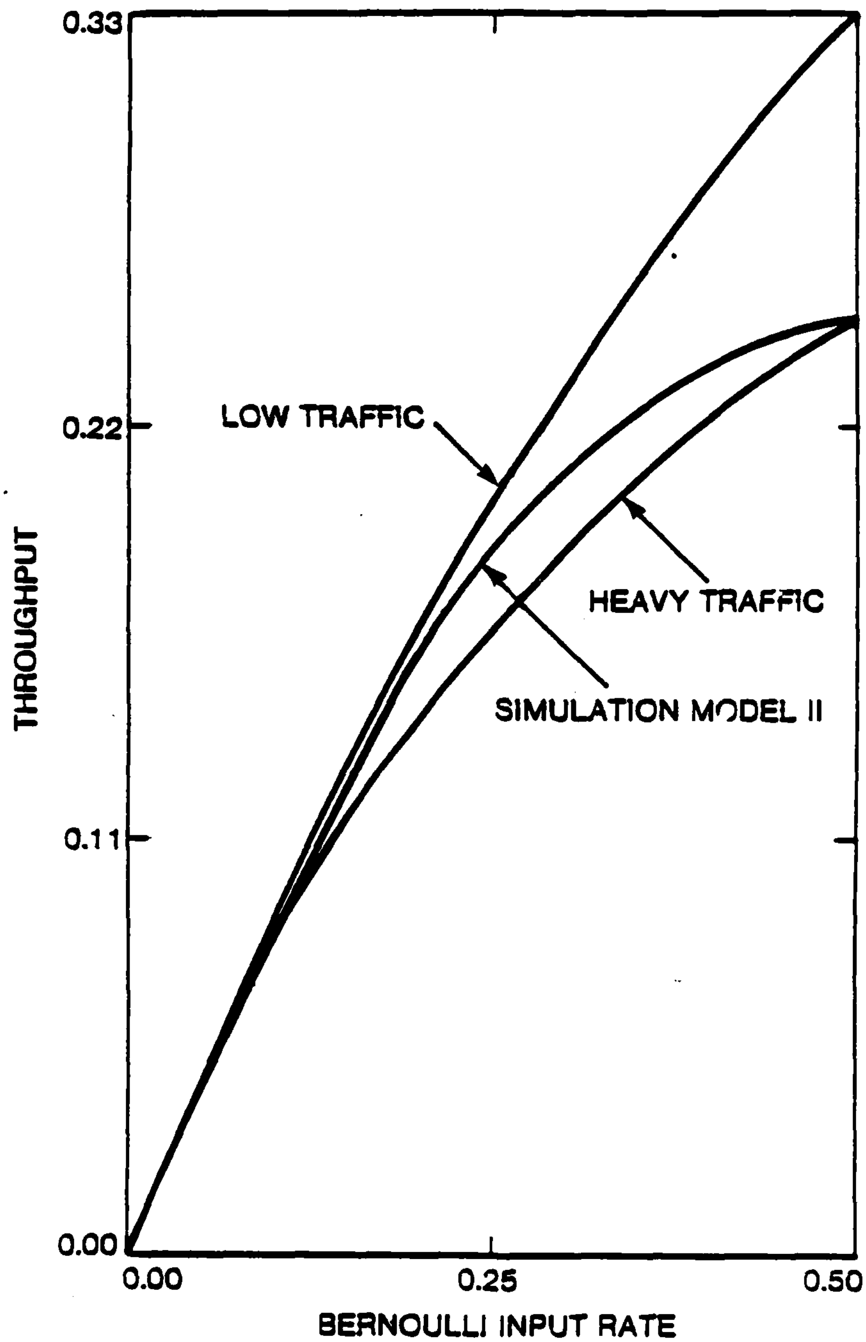


Figure 3.3-8: Heavy and Light traffic approximation of throughput-load; model II

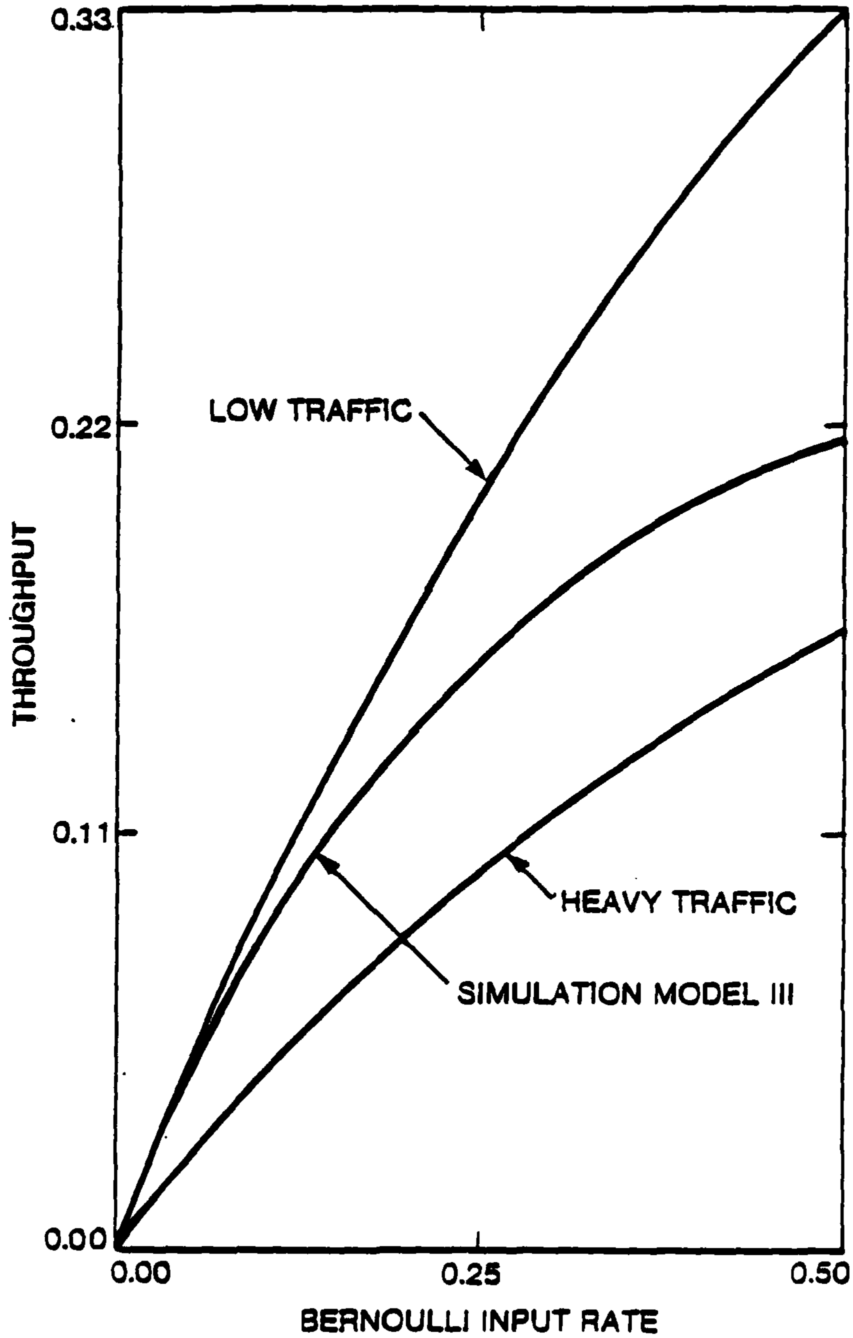
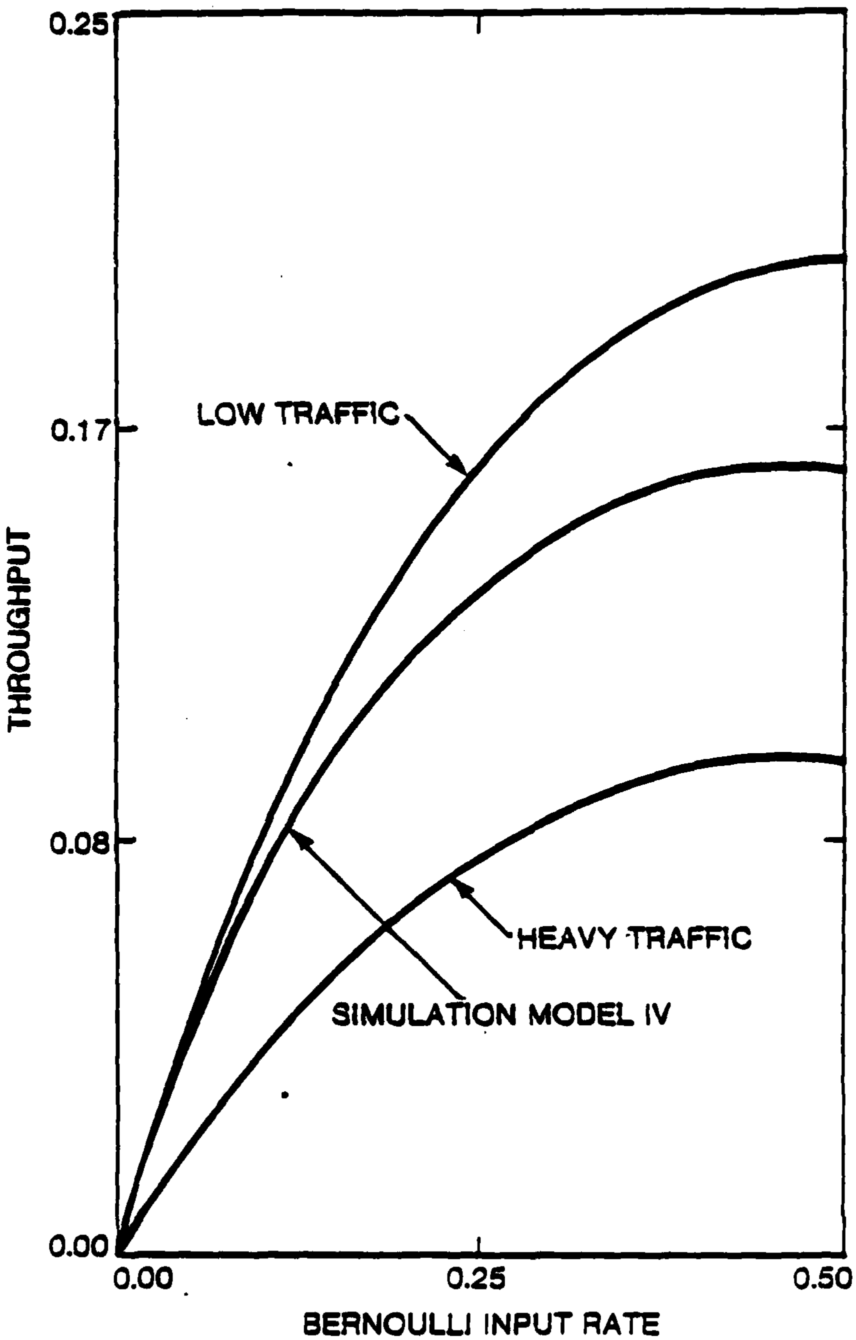


Figure 3.3-9: Heavy and Light traffic approximation of throughput-load; model III



jure 3.3-10: Heavy and Light traffic approximation of throughput-load;:model IV

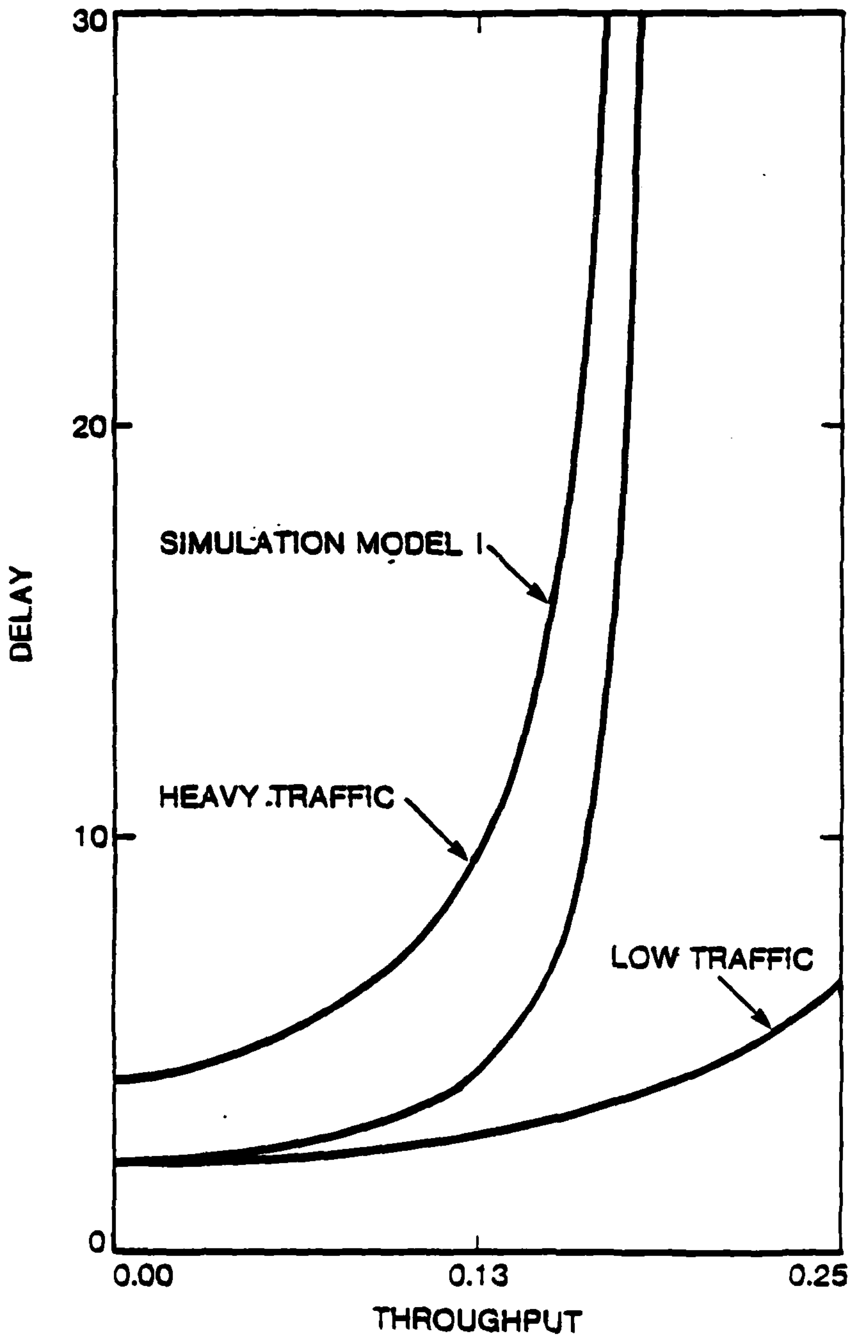


Figure 3.3-11: Heavy and Light traffic approximation of delay-throughput: model I

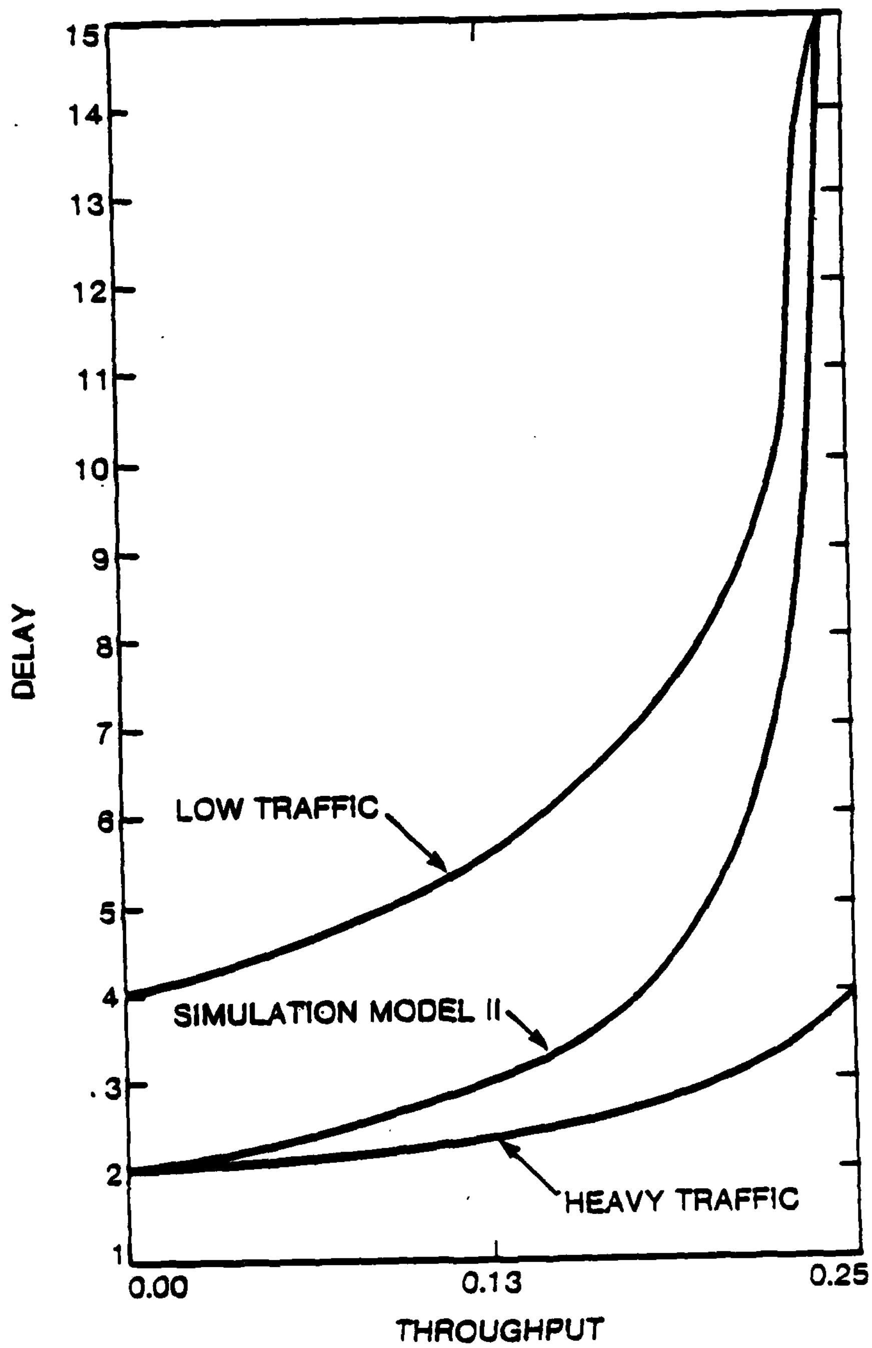


Figure 3.3-12: Heavy and Light traffic approximation of delay-throughput: model II

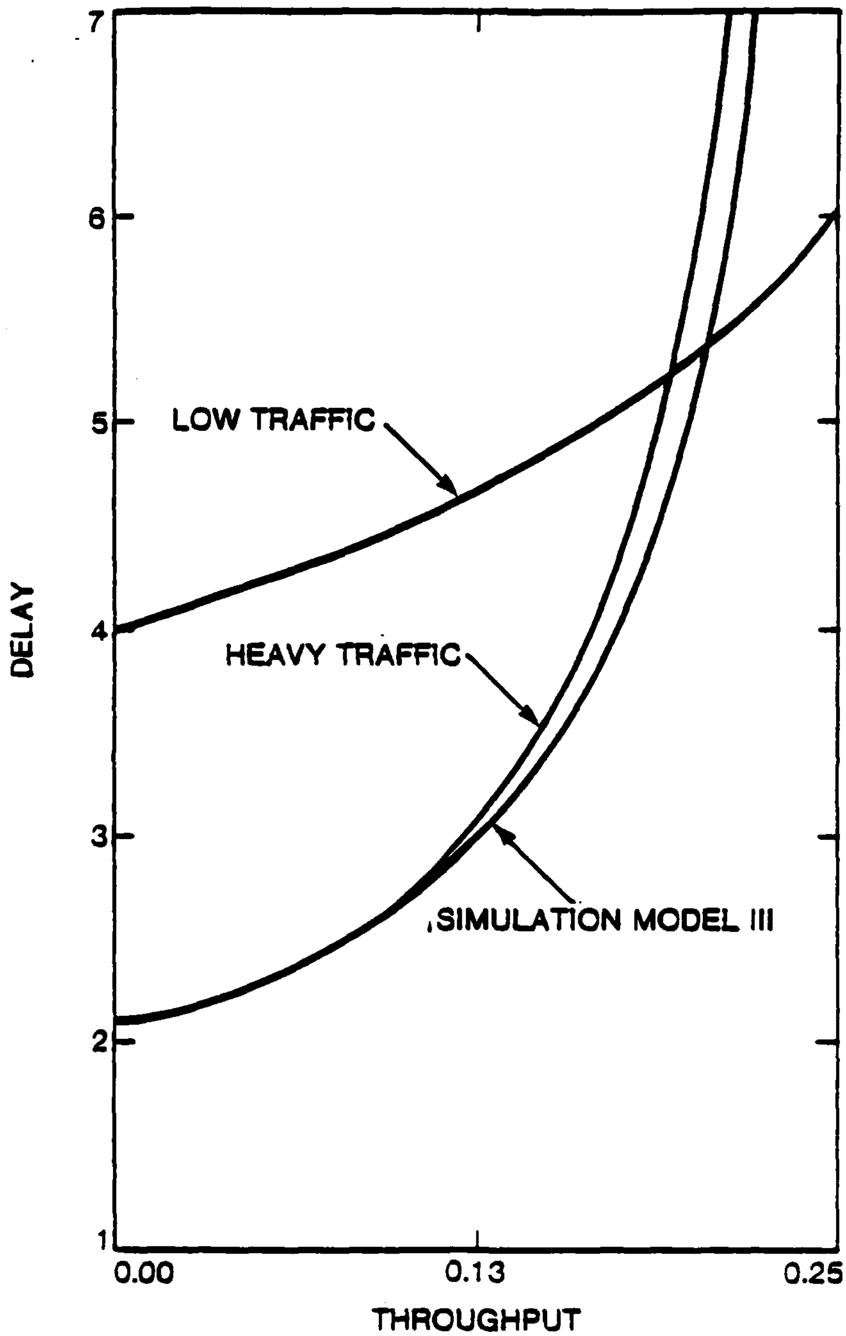


Figure 3.3-13: Heavy and Light traffic approximation of delay-throughput; model III

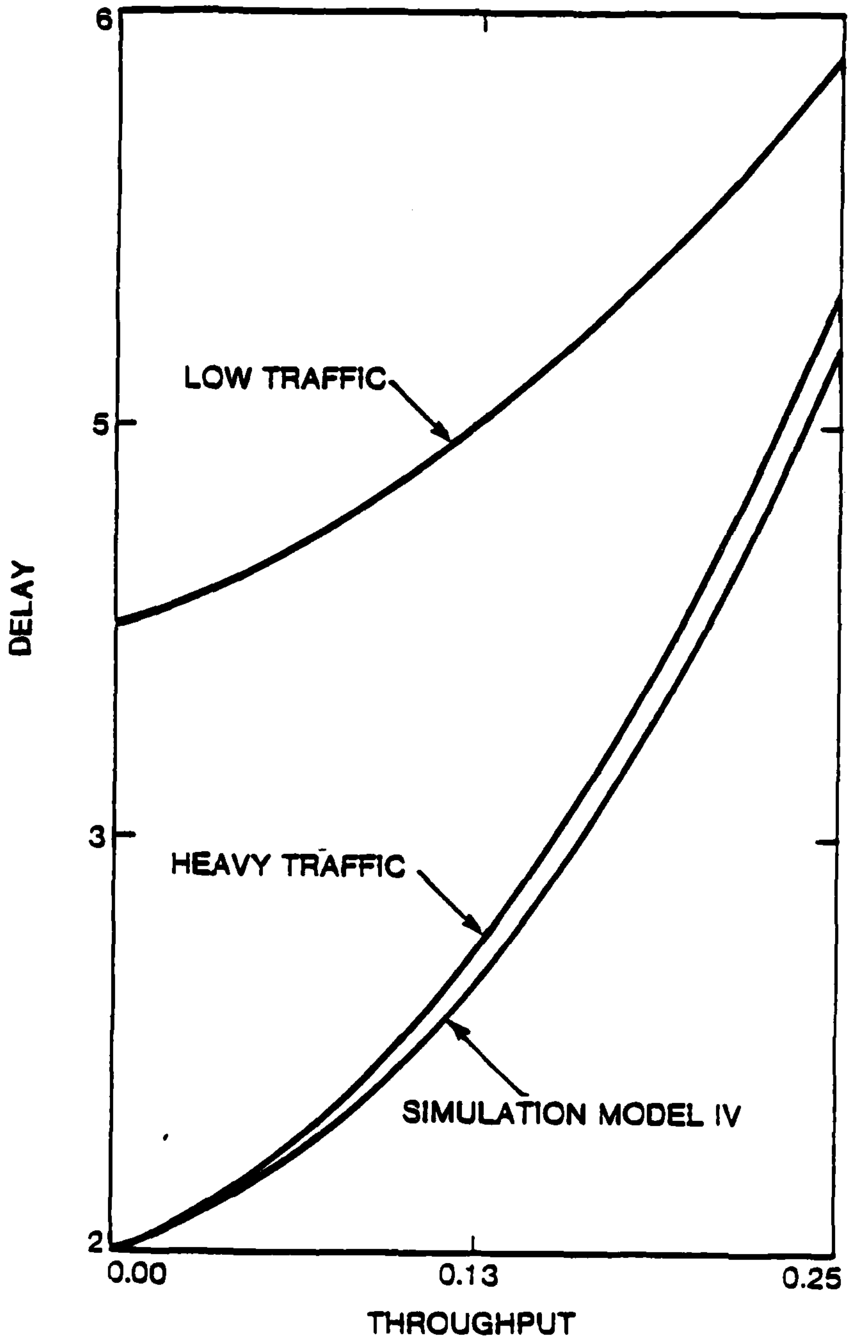


Figure 3.3-14: Heavy and Light traffic approximation of delay-throughput; model IV

3.3.3 AN EXACT SOLUTION OF THE FOURTH MODEL OR, CLOSE ENCOUNTERS OF A SINGULAR KIND

Let us recall the transition structure of the third and fourth models of section 3.1.1. The third model represents an increase in the interaction between the two PRUs. The fourth model is a "maximum interference" model, that is, incoming packets encounter each other as well as outgoing packets. The interference controls the input flow of packets into the system. In the fourth model the diagonal movements of the queueing RW were completely eliminated by the cross interference of incoming and outgoing packets; this simplification enables us to derive the solution using a relatively trivial computation. In the third model, only the north-east diagonal is left, but this is enough to render the solution an order of magnitude more difficult than the fourth model. Figure 3.3-15 depicts the transition probabilities of the fourth model.

The TDRW of the maximum-interference (fourth) model possesses a simplifying feature, namely, the relation between the boundary and the interior transitions resembles that of the projectable TDRW of section 3.4.1. The only difference is that the transition probabilities at each boundary are not exactly the projections of the interior movements but projections multiplied by constants. This TDRW is not projectable, yet the similarity to a projectable TDRW renders it solvable in terms of a simple product form of the transformed steady-state distribution. Let us proceed to derive the solution.

We reconsider the steady state equation 3.1-5 of section 3.2.1.

$$0=A^{11}G^{11} + A^{10}G^{10} + A^{01}G^{01} + A^{00}G^{00}$$

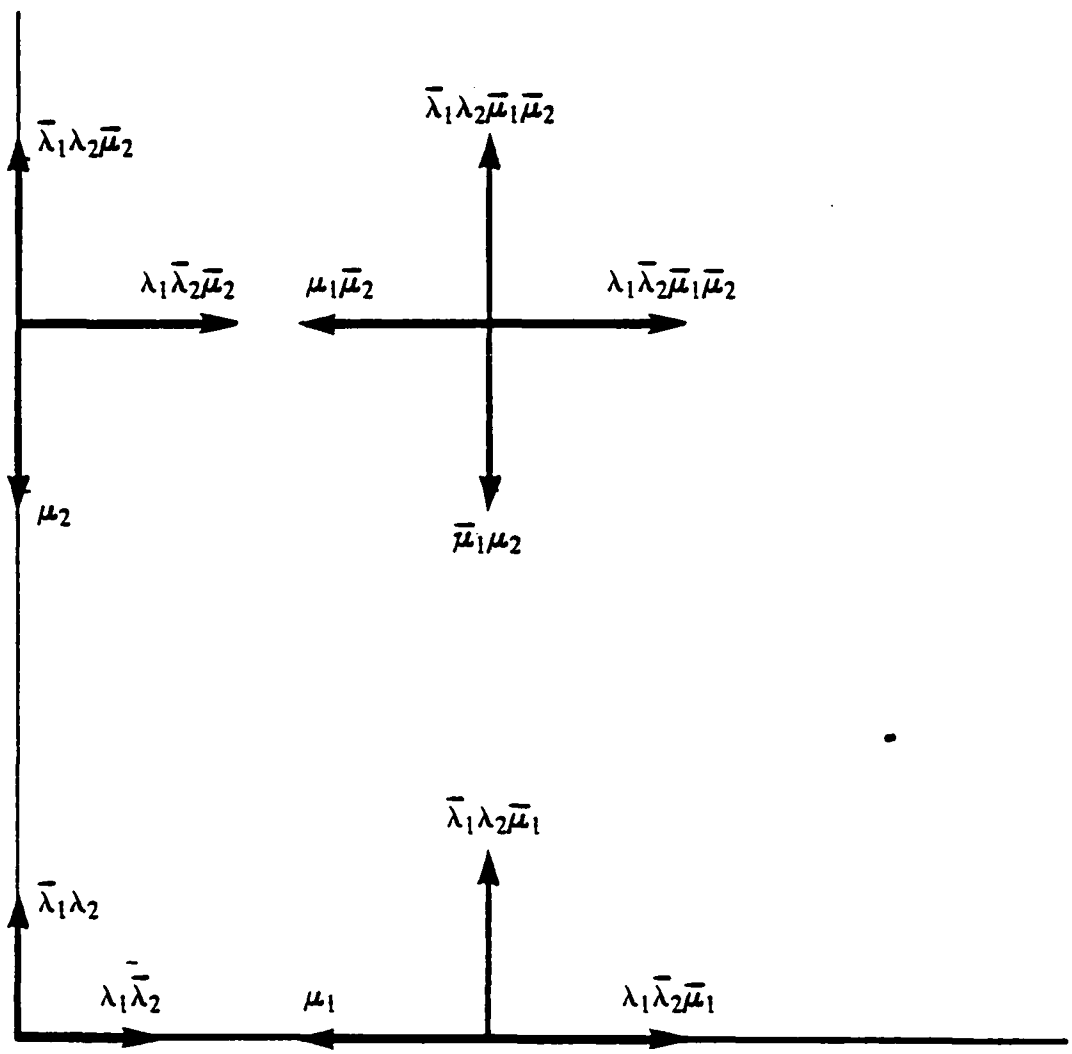


Figure 3.3-15: Transition probabilities of the fourth model

Let us scale the coefficients representing the boundary transitions and the respective transforms. That is, let us define

$$\tilde{G}^{01}(z) \triangleq G^{01}(z)/\bar{\mu}_1$$

$$\tilde{G}^{10}(w) \triangleq G^{10}(w)/\bar{\mu}_2 \tag{3.3-7}$$

$$\tilde{G}^{00} \triangleq G^{00}/(\bar{\mu}_1\bar{\mu}_2)$$

Then, the steady state equation may be rewritten in terms of the scaled transforms. The new form is similar to the equation for a projectable TDRW. It is possible to solve the scaled equation, since the respective projectable RW has no diagonal movements, and use the results to obtain the original transform. The result of the cumbersome computation is the following product form expression for the transform of the steady state distribution of the fourth model*

$$G(z,w) = \frac{[\,1/(1-\rho_1 z) - \mu_1\,][\,1/(1-\rho_2 w) - \mu_2\,]}{[\,1/(1-\rho_1) - \mu_1\,][\,1/(1-\rho_2) - \mu_2\,]} \tag{3.3-8}$$

Here $\rho_1 \triangleq \lambda_1 \bar{\lambda}_2 \bar{\mu}_1/\mu_1$ and $\rho_2 \triangleq \bar{\lambda}_1 \lambda_2 \bar{\mu}_2/\mu_2$

The expected number of packets in the buffer of PR_1, is given by

$$\bar{Q}_1 \triangleq \frac{dG}{dz}(1,1) = \rho_1 \,/\, (1-\rho_1)\,(\bar{\mu}_1 + \mu_1\rho_1) \tag{3.3-9}$$

---

*The expression is correct when $\mu_1 \neq 1$ and $\mu_1 \neq 1$. We shall consider the case $\mu_1 = \mu_1 = 1$ separately.

In particular, when the two PRUs are similar (symmetric system) the expected number in queue becomes

$$(3.3\text{-}10) \quad \bar{Q} = \frac{\rho}{(1-\rho)(\bar{\mu} + \mu\rho)}$$

When the transmission probabilities μ_i ($i=1,2$) are 1, the behavior of the system is simplified. The number of packets in each queue is at most one. The bivariate queueing process Q^t has 3 states, whose steady-state probabilities are easily computed to be $\pi(0,0)=1/(1+2\lambda\bar{\lambda})$, $\pi(0,1)=\pi(1,0)=\lambda\bar{\lambda}/(1+\lambda\bar{\lambda})$. From these steady state probabilities one can readily obtain the different performance measures.

$$(3.3\text{-}11) \quad Q = \lambda\bar{\lambda}/(1+\lambda\bar{\lambda}) \quad (\text{when } \mu=1)$$

The overall expected throughput (of both traffic streams) may be computed to be

$$(3.3\text{-}12) \quad \bar{S} = \begin{cases} 2\rho\mu\bar{\mu}/(\bar{\mu}+\mu\rho)^2 = 2\lambda\bar{\lambda}/(1+\lambda\bar{\lambda})^2 & \mu \neq 1 \\ 2\lambda\bar{\lambda}/(1+2\lambda\bar{\lambda}) & \mu = 1 \end{cases}$$

Using Little's result we may compute the expected delay of a packet

$$(3.3\text{-}13) \quad T = \begin{cases} (1+\lambda\bar{\lambda})/(\mu-\lambda\bar{\lambda}\bar{\mu}) & \mu \neq 1 \\ 1 & \mu = 1 \end{cases}$$

Let us note that the expected delay decreases as the probability of

transmission increases towards 1. Moreover, when the probability of transmission μ assumes the value 1 a *discontinuous* improvement of performance occurs; the expected throughput exhibits a jump increase and the expected delay has a jump decrease. This discontinuity will soon be elaborated. The delay-throughput performance of the transmission policy μ=0.5 is compared in Figure 3.3-16 with the results of simulation.

Figure 3.3-17 depicts the discontinuous behavior of the delay T as a function of the transmission policy. The "rude" choice of the transmission policy to be μ=1 causes a singular decrease in the delay.

The optimality of the rude policy is intuitively clear. Indeed, once a packet enters the system it is guaranteed immediate uninterrupted service. A new packet is permitted into the system iff the system is empty and no other packet tries to enter. After entry, the expected delay is exactly one slot and no channel waste in collisions or empty slots occurs.

The employment of a rude policy results in a *phasing* of the arrivals and transmissions. The system exhibits phased service cycles. An arriving packet is delivered from the second hop (the terminal level) to the first hop (the repeater level); then it is delivered to the station. At the end of each cycle the system is ready for the next service cycle. Figure 3.3-18 depicts a typical phased propagation of packets. Through phasing, the system obtains the best performance which is possible for any two hop system, namely, delay of one-slot per accepted packet (minimum possible) and, in the case of "maximum interference", maximal throughput possible (as much as the limits of interference permit).

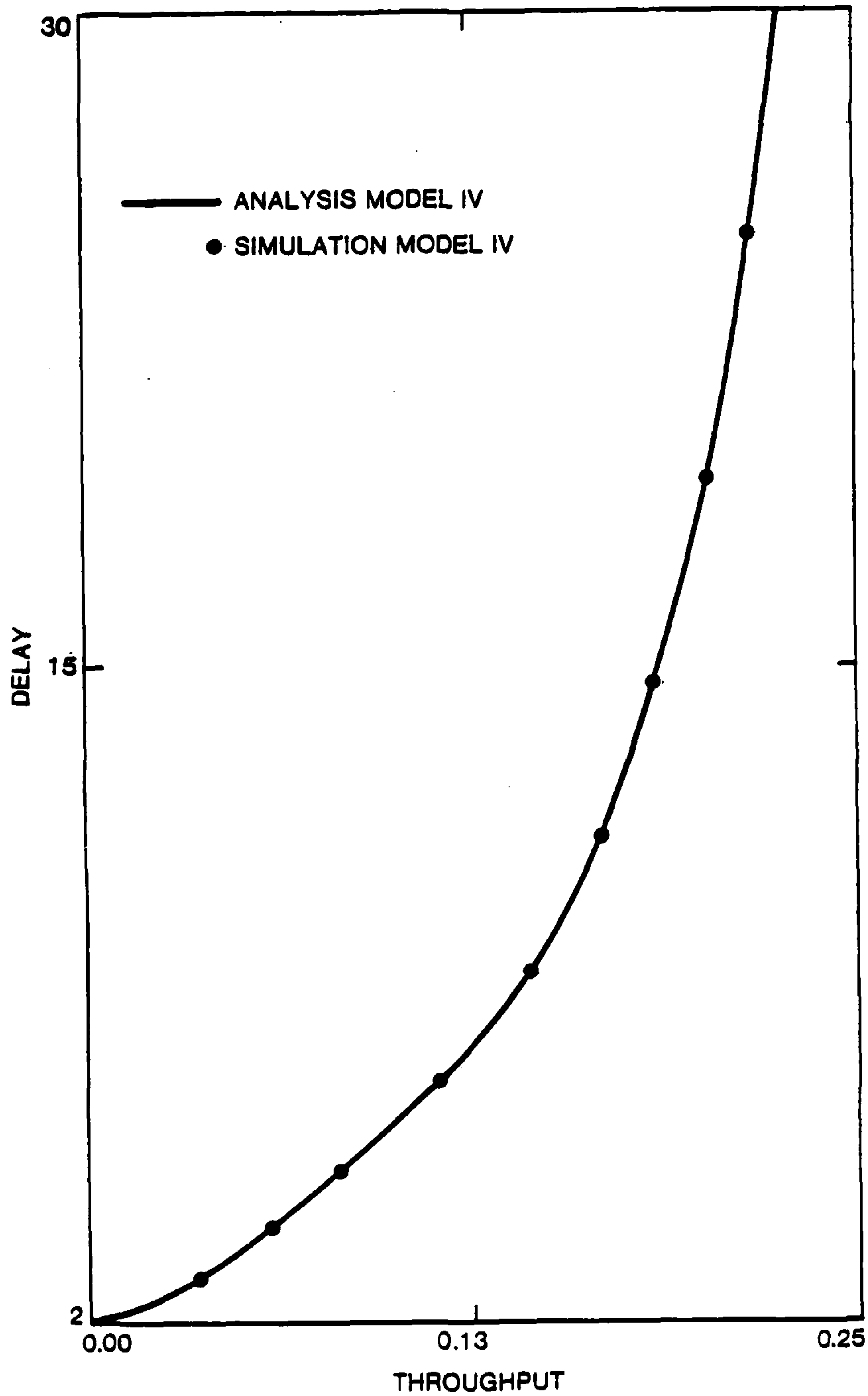


Figure 3.3-16: Delay-throughput performance of the "maximum interference" model

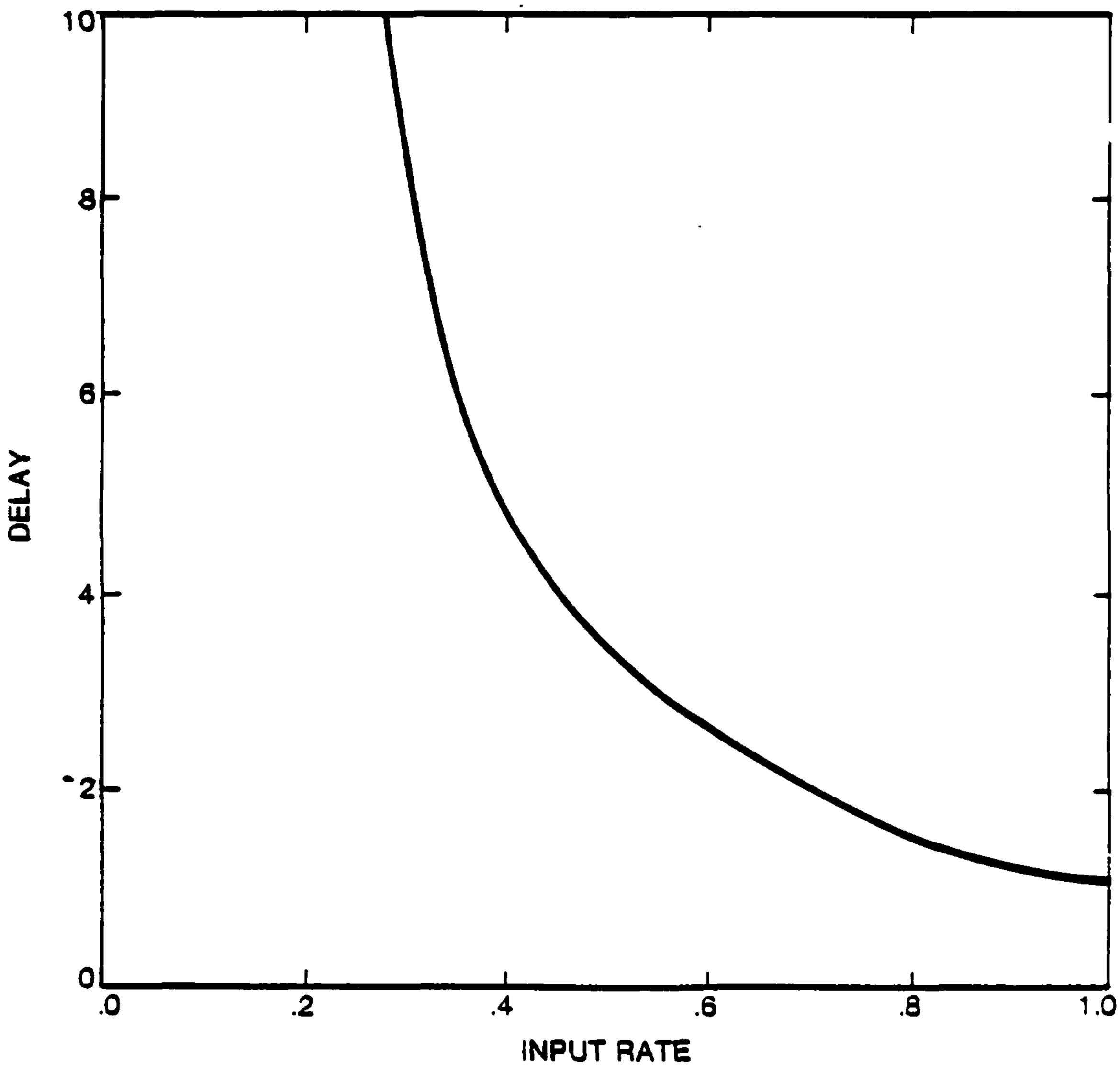


Figure 3.3-17: Discontinuous decrease in delay for the "rude" policy

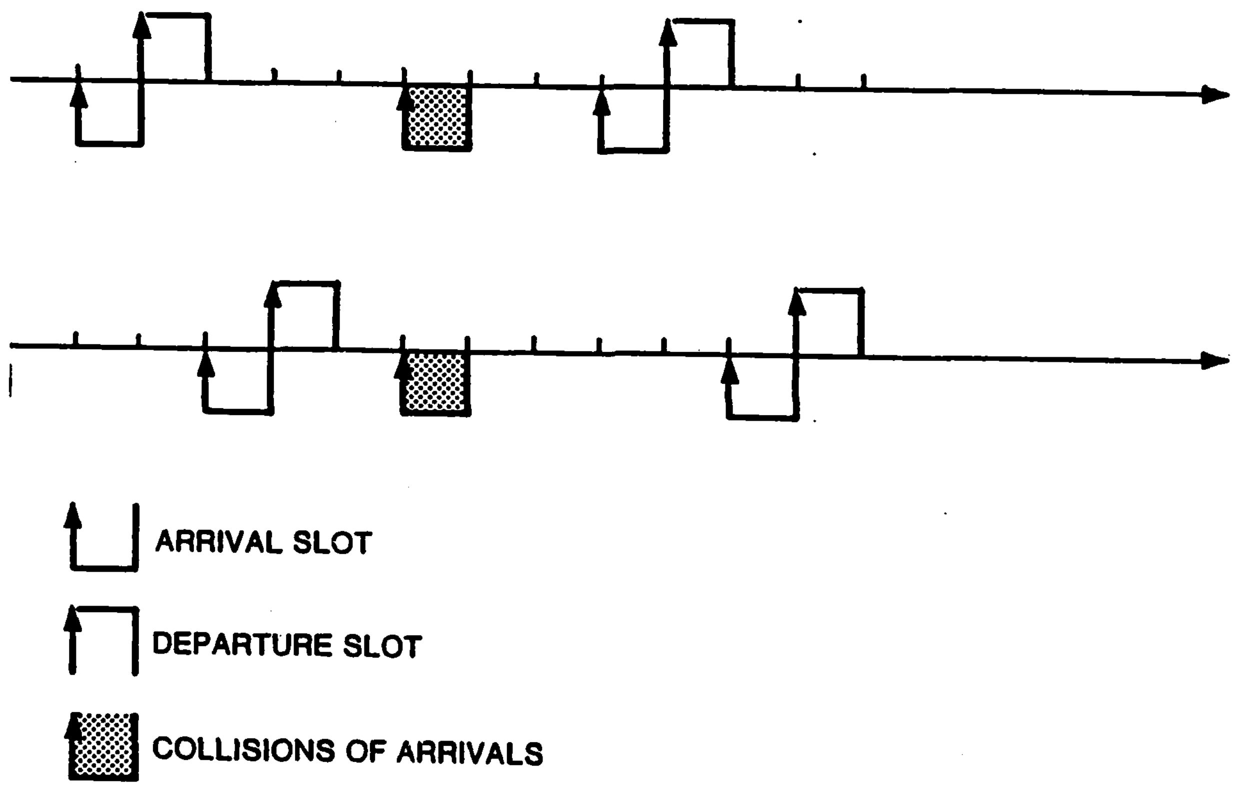


Figure 3.3-18: Phased propagation in a "rude" maximum-interference model

The surprising effect is the singularity of the rude behavior. The discontinuity of performance arises because of the discontinuous change in the transition structure of the queueing RW Q^t. We shall have further occasions to meet singular systems in chapter five. Singular combinatorial structures of interference are explored in section 5.3. There again singular improvements of performance occur when a rude policy is selected. The cross interference is utilized to obtain phasing of packet movements through the network. The phasing obtains perfect scheduling of the transmissions.

What is the reason for the difference in performance when the transmission policy is selected to be μ=0.999999 and when it is selected to be μ=1 ? The answer is simple. The rude policy precludes the possibility that the two queues will ever be busy at the same time. The policy μ=0.999999 renders the event "both PRUs are busy" highly improbable yet possible. When both PRUs finally become busy they will keep colliding with each other for a very long period of time. Therefore the expected delay exhibits a discontinuous decrease when μ increases from 0.999999 to 1.

Another surprise is the sensitivity of the two buffered PRUs problem to small changes in the interference structure. Indeed neither the first, second or third models even admit a rude policy. Nor is it possible to solve those models in terms of a simple computational procedure such as the one above. A small change in the combinatorics of interference may lead to a totally different solution.

Let us compare the Slotted Aloha scheme of channel sharing in the fourth model to a deterministic allocation of the channel. We choose to split the

channel between the two PRUs on an equal basis. That is, each traffic stream gets "half of the channel" (say, through a frequency division) for it's exclusive use. Nevertheless the incoming traffic to each PRU shares the same channel with the outgoing traffic. Each PRU uses a coin tossing scheme to resolve collisions between incoming and outgoing packets.

The queueing processes at the two PRUs are independent. The queue length at each PRU performs a one-dimensional, positive, integer RW. The transition structure of this RW is depicted in Figure 3.3-19. Note: the length of slots is now twice the length of the slots in the case of shared channel. The delay and throughput expressions need to be scaled accordingly.

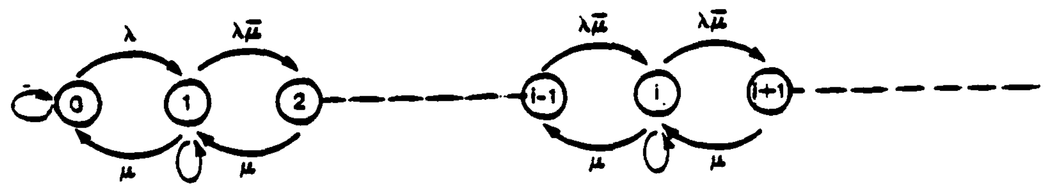


Figure 3.3-19: Transition diagram for the "split" channel model

Let us specialize the solution of section 3.4.1, for the general RW with a similar transition structure. The transform of the steady state distribution for the number in queue is given by:

$$G(z) = \frac{1/(1-\rho z) - \mu}{1/(1-\rho) - \mu} \qquad (3.3\text{-}14)$$

Where

$$\rho \triangleq \lambda\bar{\mu}/\mu \qquad (3.3\text{-}15)$$

The expected number in each queue becomes

$$\bar{Q} \triangleq G'(1) = \rho / \left[(1-\rho)(\bar{\mu} + \mu\rho) \right] \qquad (3.3\text{-}16)$$

The overall expected throughput (of both streams) becomes

$$S = \lambda/(1+\lambda) \qquad (3.3\text{-}17)$$

The expected delay experienced by packets is

$$T = 2/(\mu - \lambda\bar{\mu}) \qquad (3.3\text{-}18)$$

The best transmission policy is again the rude policy. It is always better to transmit the packet at hand than to keep silent hoping that an arriving packet will make use of the silent slot. The throughput does not depend upon the transmission policy. However the rude policy does not exhibit a discontinuous jump. Indeed as μ approaches 1 the performance of the PRU smoothly approaches that of the rude policy.

Let us reconsider the expressions for the transforms of the respective distributions, 3.3-14 and 3.3-8, of the one and two dimensional queueing processes. Let the transmission policy converge to the rude policy. The two transforms converge to

$$\lim_{\mu \to 1} G(z) = (z+\lambda)/(1+\lambda) \qquad (3.3\text{-}19)$$

and

$$(3.3\text{-}20)\quad lim_{\mu \to 1}\, G(z,w) = \frac{(z + 1/\lambda\bar{\lambda})\,(w + 1/\lambda\bar{\lambda})}{(1 + 1/\lambda\bar{\lambda})}$$

respectively.

While the first limiting transform represents the transform of the limiting steady state distribution of the one dimensional queue; the second limiting transform does not represent the limiting distribution of the two queues. Indeed, it assigns a positive mass to the state (1,1) which is clearly never obtained under the rude policy. The source of the disparity is the discontinuous change of performance when the transmission probability is raised from 0.999999 to 1.

Let us now compare the performance of the two configurations, that is, two completely interfering streams, sharing the same channel and two streams, using a deterministic splitting of the channel to avoid interference (but sharing too). We consider the two configurations at their best, i.e., when rude policies are used. respective throughput functions 3.3-12 and 3.3-17 are plotted against the Figure 3.3-20 depicts the ratio of the throughputs 3.3-12 over 3.3-17 as a function of the input rate λ. The clear result is that rude sharing is to be prefered to channel splitting for all reasonable input rates, i.e., $\lambda < 1/2$. So much as far as throughput is concerned.

Let us compare the delay performance of the two configurations. Fix the transmission policy to be μ. The ratio of the expected delay functions 3.3-13 over 3.3-18 is given by

$$(3.3\text{-}21)\qquad (1/2)(1 + \lambda\bar{\lambda})\,[\,1 - \lambda^2\bar{\mu}/(\mu - \lambda\bar{\lambda}\bar{\mu})\,]$$

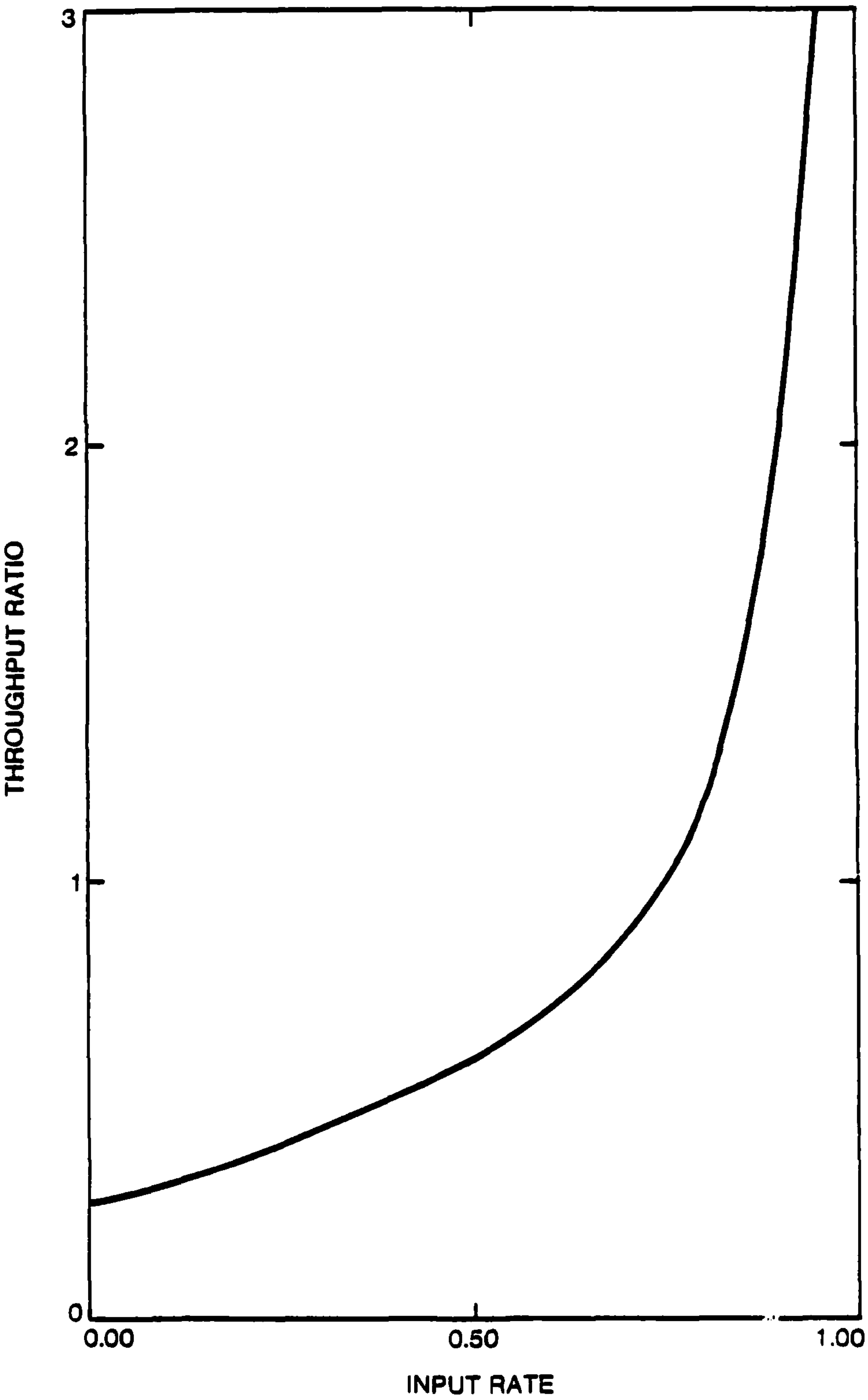


Figure 3.3-20: Ratio of throughputs for unsplit/split channel, vs. load

This expression is always smaller than 1. Thus sharing is to be preferred. When the input rate approaches 0, the shared channel is twice as fast as the split channel. Figure 3.3-21 depicts the ratio of delays as a function of the input rate.

Finally let us note that close hearing encounters for models with more than two PRUs exhibit similar singularities. Indeed, *if arrivals are blocked by transmissions and if simultaneous arrivals are precluded through a "maximum interference" model, then the exact solution of the queueing MDRW is given in a product form and the rude policy is optimal and singular.*

To conclude the discussion of the solution for models with close encounters between packets due to a "maximum interference" hearing we have:

- Sharing is better than deterministic allocation.

- Multi-dimensional systems may exhibit dichotomies, such as the singularity of the rude policy.

- Rudeness, when possible, may obtain perfect scheduling by employing cross interference to create phased service cycles.

- The behavior of multi-dimensional systems may be very sensitive to changes in the combinatorics of interference.

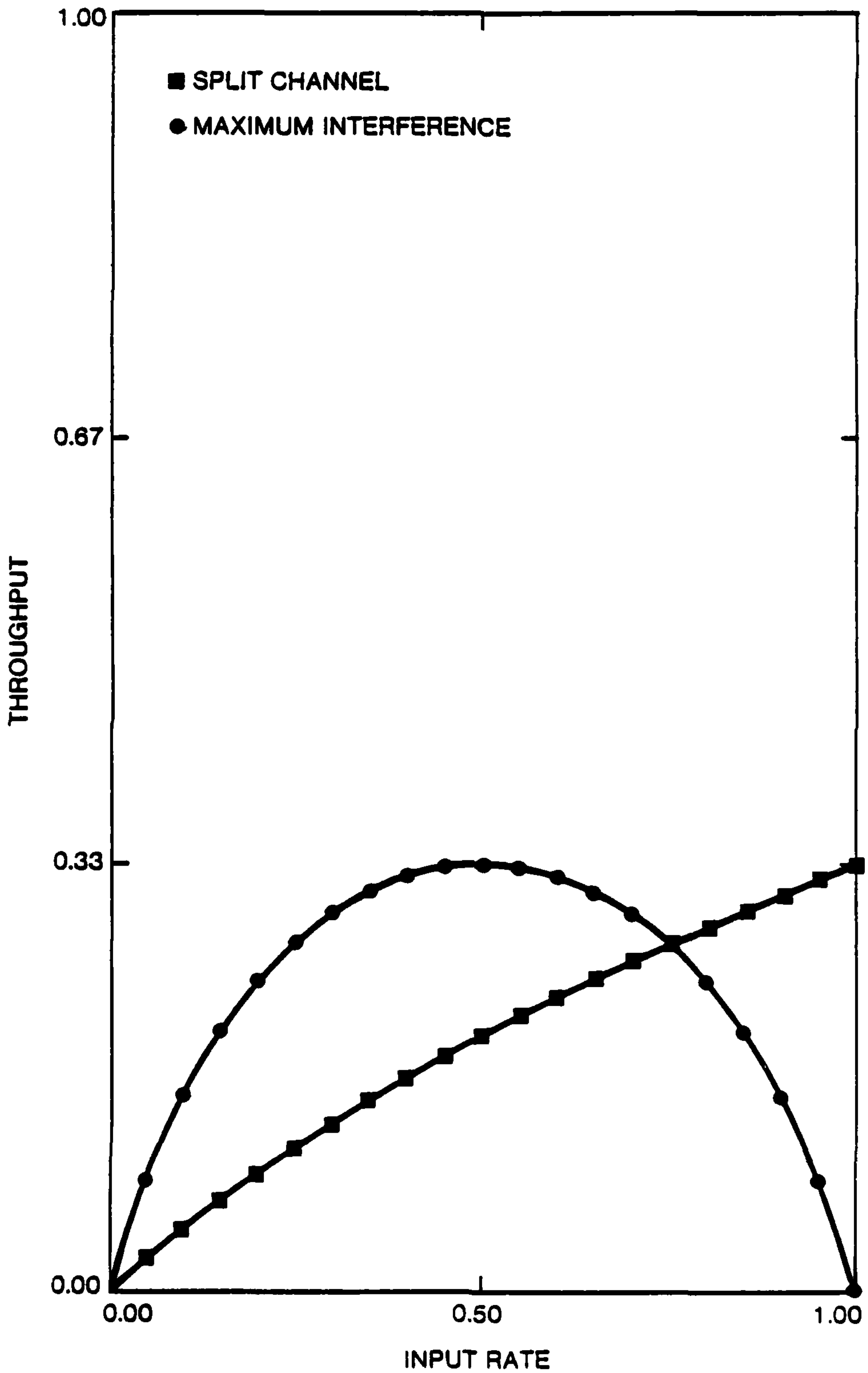


Figure 3.3-21: Ratio of delays for unsplit/split channel, vs. load

4. A WIENER-HOPF TECHNIQUE TO SOLVE THE TDRW

In this chapter we develop a Wiener-Hopf technique to solve the general integer, nearest neighbor, positive TDRW. We shall first describe the solution technique from a geometric point of view, then demonstrate some applications starting from easier TDRWs to complex cases. From a conceptual point of view, the problem is reduced to that of a Wiener-Hopf factorization over compact Riemann surfaces of Genus 0 or 1. From a computational point of view, the solution is usually very complex.

Perhaps, at this point, it is worthwhile to comment on the possibility of obtaining a closed-form solution. As we shall see, even the simplified cases of TDRW that may be solved in terms of "known" functions, lead us to the boundaries of our small dictionary of "known" functions. Moreover, the algorithms suggested by the closed-form expressions are usually more complex than approximate numerical solutions. It seems that the very objective of deriving closed form solutions is, most probably, unattainable, unless of course we expand our dictionary to "know" new functions. In our case, it is necessary to "know" new classes of Theta functions and integrals of Theta functions.

The main objective of our investigation is, thus, not necessarily to develop closed form solutions, but to develop a better geometric and algebraic understanding of the problem. As far as computations are concerned, the "solution" proposed should mainly serve as a basis for a set of approximation schemes, similar to heavy traffic "one pole approximations" in one dimensional queueing theory. This as well as other interesting problems are beyond the

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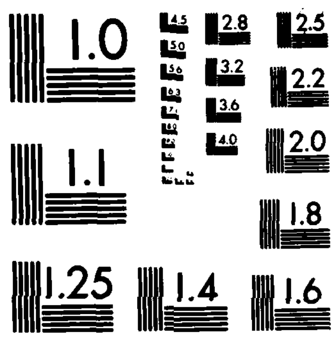

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scope of this dissertation and are left for future research.

The Wiener-Hopf technique is extensively discussed in [NOBL56]. The relation of the Wiener-Hopf technique to the behavior of bounded Markovian processes is described in [FELL64, KEMP61]. The Wiener-Hopf technique plays a major role in queueing theory. Particular applications to queueing processes can be found in [COHE76, KING63, KLEI75]. Singular integral equations and the Riemann-Hilbert problem are presented in [GAKH63, MUSH48] these are intimately related to our problem, since it is possible to represent the problem of computing the transform of the steady-state distribution as that of solving singular integral equation whose kernel is the Green-Function of the TDRW [KEIL65].

4.1 THE GEOMETRY OF THE TRANSFORMED STEADY-STATE RELATIONS

4.1.1 REFORMULATING THE PROBLEM

Let us recall the steady state equation for the general nearest neighbor TDRW (equation 3.2-5 of section 3.2.1):

$$(4.1\text{-}1)\qquad 0=A(z,w)G^{11}(z,w)+A^{10}(z,w)G^{10}(z)+A^{01}(z,w)G^{01}(w)+A^{00}(z,w)G^{00}$$

(The superscripts of the first coefficient A^{11}, are dropped for convenience).

This equation in the four unknown: $G^{11}(z,w)$, $G^{10}(z)$, $G^{01}(w)$ and G^{00}, contains all the information about the steady state behavior of the RW. That is, the RW is stable iff equation 4.1-1 possesses a unique solution, such that

(4.1-2) $G^{11}(z,w)$, $G^{10}(z)$ and $G^{01}(w)$ are analytic in the respective disks

$$(4.1\text{-}3)\qquad G^{11}(1,1)+G^{10}(1)+G^{01}(1)+G^{00}=1$$

The determination of the solution of 4.1-1 which satisfies the analyticity conditions 4.1-2 and the normalization condition 4.1-3 is our major problem. Let us note in passing that once we obtain a solution of 4.1-1 subject to the analyticity condition 4.1-2, it is possible to satisfy the normalization condition 4.1-3 trivially. Therefore we shall assume for the time being that G^{00} has been factored out from the equation 4.1-1.

The main idea which we pursue is to consider the algebraic curve:

$$(4.1\text{-}4)\qquad 0= A(z,w) = z^2p(w)+zq(w)+r(w) = w^2s(z)+wt(z)+v(z)$$

which we have called the *characteristic curve* of the RW. On this curve the steady state equation reduces to:

$$(4.1\text{-}5) \qquad 0=A^{10}(z,w)G^{10}(z)+A^{01}(z,w)G^{01}(w)+A^{00}(z,w)G^{00}$$

We call this functional equation: the *boundary equation*. The idea is to solve the boundary equation first, then use the results to obtain a solution for the steady state equation 4.1-1.

4.1.2 THE CHARACTERISTIC CURVE

Let us consider the characteristic curve defined by equation 4114 of the previous section. Let $S \triangleq \{(z,w) | A(z,w)=0\}$ denote the Riemann surface of the curve. We shall use the letters P, Q, R etc... to denote points on S. The surface S represents a cover of both the Z and the W spheres, with the natural projections [SING55]:

$$h_1 : S \dashrightarrow Z \qquad (4.1\text{-}6)$$

$$h_1(z,w) \triangleq z$$

The projection $h_2 : S \dashrightarrow W$ is defined similarly.

The equation defining S is in general bi-quadratic. Therefore the *genus* [SIEG69] of the surface S is in general 1 (i.e., the surface is topologically equivalent to a torus). However, in some special cases the genus degenerates to 0 and S can be represented as a sphere in terms of rational or periodic functions. Such degenerate cases arise if the transitions of the TDRW in the interior are restricted to one side of a hyperplane.

As an example of degeneracy consider two PRUs using an *isarithmic*-like flow-control policy whereby the total number of queued packets in the system is permitted to grow iff one PRU is idle. The *isarithmic policy* can be implemented in a number of ways. One possibility is that a PRU accepts a new packet iff its fellow PRU is idle, or has just used the channel successfully. The transition structure of the isarithmically controlled TDRW is depicted in figure 4.1-1. Another TDRW of genus 0 arises from the problem of "shortest queue"

[KING61, McKEA76]]. In the following section we derive a closed-form solution to the general TDRW of genus 0. Let us proceed now, to examine the more general case.

In general S is a surface of genus 1. It represents a four branched, two sheeted covering of the Z and W spheres. A concrete form of S can be generated by pasting two copies of the Z sphere along the two branch cuts. The resulting surface is a torus. The equation defining S, being bi-quadratic, to each value of z there are two points on S, (z,w_1) and (z,w_2), which solve the characteristic equation. The permutation of the two solutions corresponding to a single value of z is denoted T_1. The operator T_1 is an automorphism of S which is projected onto the identity transformation of Z. Such a map is called a *cover transformation* [SING55]. Indeed, the group $\mathfrak{R}_1 \triangleq \{I, T_1\}$, where I is the identity on S, represents all cover transformations of (S,h_1). (Note that $T_1^2=I$.) Similarly one defines T_2 to be the cover transformation which permutes the two solutions of the characteristic equation for z, given a value of w. The group $\mathfrak{R}_2 \triangleq \{I, T_2\}$ constitutes the cover transformations of (S,h_2).

The maps T_i can be figuratively described as flipping the two sheets of S. Figure 4.1-2 depicts the relations between S, Z, W and the operation of the automorphisms T_i.

The torus S may be further covered by a *universal covering* [SING55], depicted in figure 4.1-2 as the plane U. The plane U is tesselated into an infinite number of period parallelograms. Each parallelogram represents a copy of the torus S, which has been dissected along two simple closed curves that cross each other at a single point, such that one curve encloses one of the branch cuts and the other

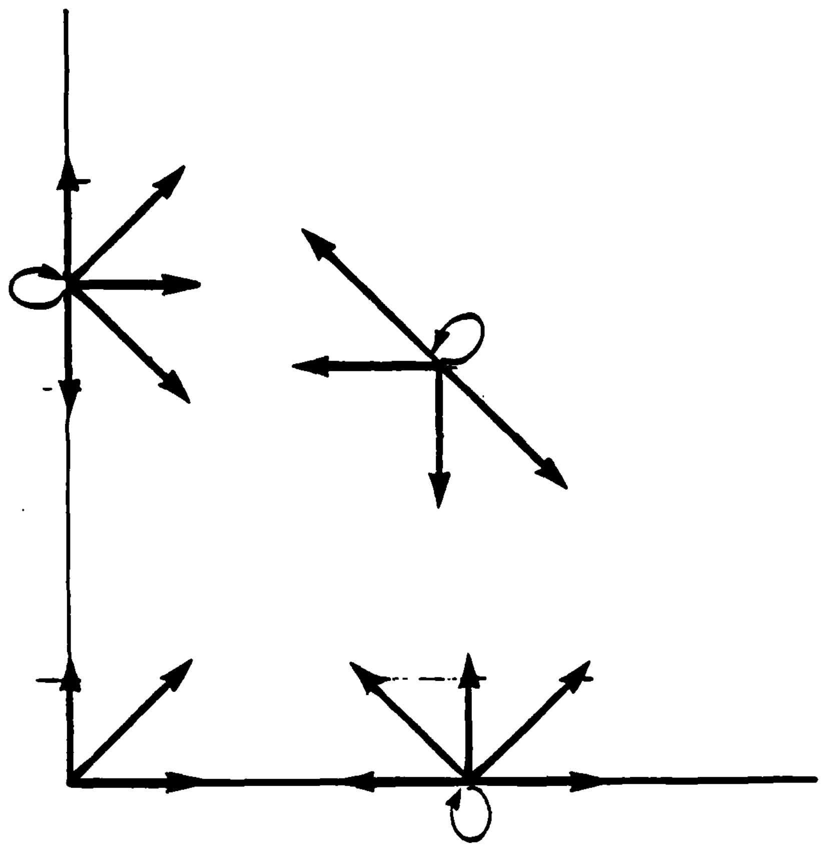

Figure 4.1-1: Transition structure of the "Isarithmic" system

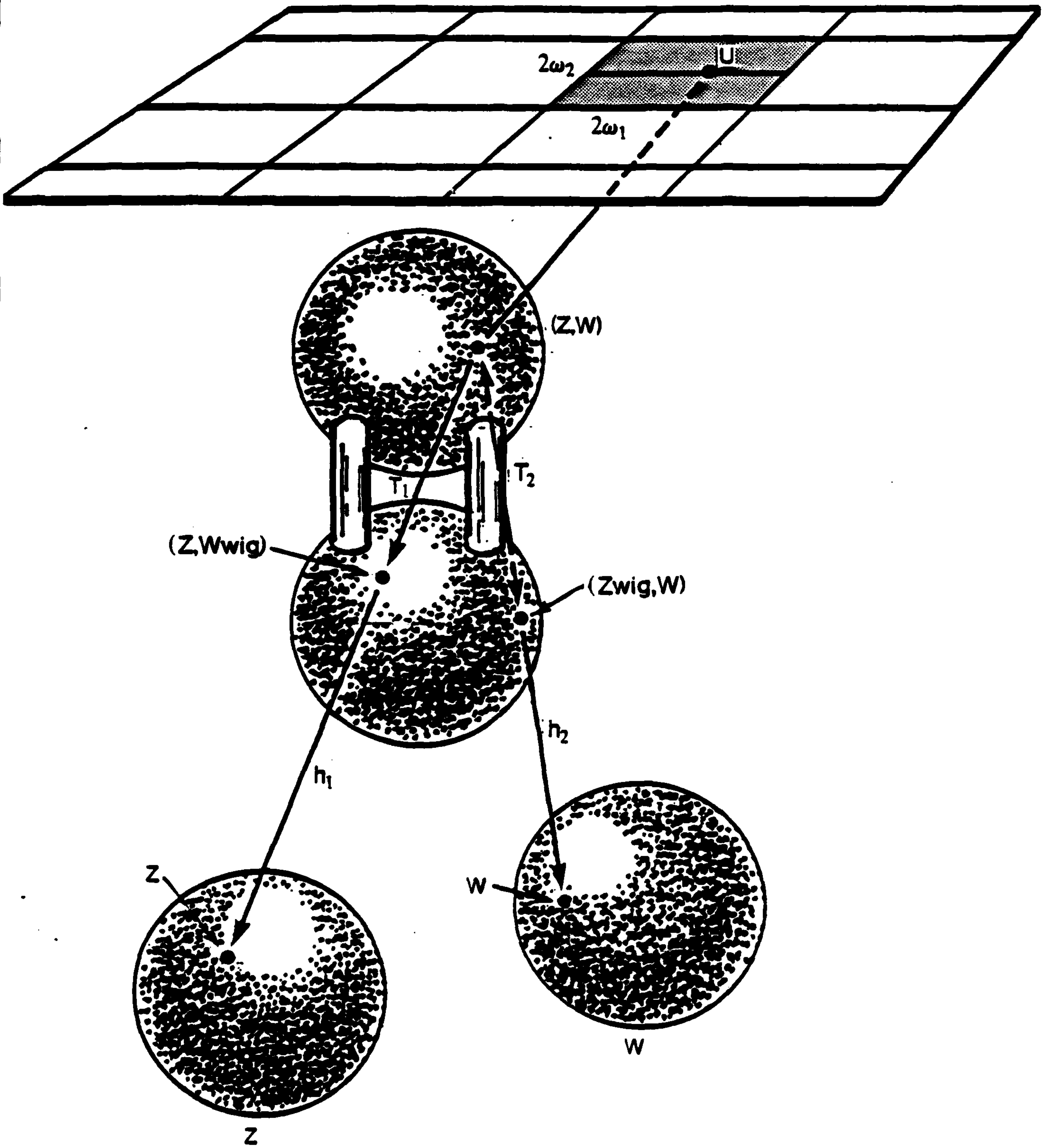

Figure 4.1-2: The geometry of the characteristic curve

curve crosses the second branch cut. Such curves are called *homology basis* for the surface S [SING55]. The plane U is equivalent to the torus once we identify points whose difference is an integral linear combination of the two basic periods $2\omega_1$ and $2\omega_2$. The last pair of complex periods forms a basis for the lattice of parallelograms in the U plane.

Finally, the plane U is a uniformizing plane for the curve S. That is, it is possible to represent z and w as uniform (one-valued meromorphic) functions over the plane U so that $A(z(u),w(u))=0$. The functions z(u) and w(u) are doubly periodic w.r.t. the periods $2\omega_1$ and $2\omega_2$. In other words, the whole process of covering provides a parametric representation of the characteristic curve in terms of *elliptic* (doubly periodic and meromorphic) functions. The actual computation of functions which uniformize the characteristic curve will be carried out later. In this section we wish to mask the computational details, giving a rudimentary description of the solution.

When the characteristic curve degenerates into a curve of genus 0, the uniformization process is simplified and may be carried out in terms of rational or trigonometric (simply periodic) functions. The computational details will soon be presented.

4.1.3 THE GEOMETRY OF THE ANALYTICITY REGIONS

Let us consider the unit disks of the Z and W planes, $D_z(1)$ and $D_W(1)$ respectively. Both regions may be lifted to any of the covering surfaces which represent the characteristic curve S. Since we intend to solve the boundary functional equation 4.1-5 on the surface S, subject to analyticity conditions in the respective regions 4.1-2, it is imperative that the structure of these regions be considered.

Let us consider the characteristic equation when w is restricted to the boundary ∂D_W, i.e., $|w|=1$. On this curve, the following inequality holds for z on ∂D_z:

$$(4.1\text{-}7) \qquad |A(z,w)+zw| = \left| \begin{bmatrix} w^2 & w & 1 \end{bmatrix} \begin{bmatrix} \alpha_6 & \alpha_5 & \alpha_4 \\ \alpha_7 & \alpha_0 & \alpha_2 \\ \alpha_8 & \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} 1 \\ z \\ z^2 \end{bmatrix} \right| \leq 1 = |zw|$$

The equality in the leftmost inequality holds iff $w=z=1$. Therefore for a fixed $w \neq 1$ such that $|w|=1$, the functions (of z) $g(z) \triangleq A(z,w)+zw$ and $f(z) \triangleq -zw$ satisfy on ∂D_z the inequality $|g(z)| < |f(z)|$ which implies (using Rouche's theorem [FORS65]) that $A(z,w)=f(z)+g(z)$ has inside D_z the same number of zeros as f(z). Thus, when $|w|=1$ and $w \neq 1$, A(z,w) *has exactly one solution for z inside the unit disk and one outside*. For $w=1$ it is easy to check that the two solutions for z are, $z=1$ and $z=1/\rho_1$ ($\rho_1 \triangleq p(1)/q(1)$).

Therefore the W unit circle corresponds (under the map $h_1 o h_Z^{-1}$) to two simple closed curves in the Z plane; one which lies inside D_z and the other outside D_z. The point $w=1$ corresponds to two points $z=1$ and $z=1/\rho_1$. Since the

two curves do not cross each other or the unit circle ∂D_Z, the respective 4 curves on S corresponding to the two unit circles must all be homologous to the same element of the homology basis of S.

The parameter ρ_1 has an interesting practical meaning: it is precisely the ratio between the average drift to the right and the average drift to the left, when the RW is in the interior region. This ratio represents a *utilization factor* of the queueing process described by the horizontal axis. Namely, ρ_1 is the ratio between the average rate of arriving customers and the average rate of service, for the horizontal queue, when both queues are busy. Recall that in the case of one queue, a necessary and sufficient condition for stability is that $\rho_1 < 1$. Henceforth, we restrict our attention to the case of *strong stability* i.e., $\rho_1 < 1$ and $\rho_2 < 1$.

The geometrical structure of the analyticity regions is now obvious. The two unit disks are described on the torus S in Figure 4.1-3. We may choose to call the two curves which meet at the point (1,1) of S: Γ_1 and Γ_2 respectively. The other two curves are given by $T_1(\Gamma_1)$ and $T_2(\Gamma_1)$ respectively.

It is now easy to visualize the operation of the automorphisms T_1 and T_1, vis-a-vis the respective unit disks. Figure 4.1-3 depicts the full picture. The curve b_1 represents the branch cut interior to the unit disk of Z, which was used to generate the torus S. This branch cut remains invariant w.r.t. the permutation of the two sheets. Indeed, the map T_1 only reflects the upper lip of the cut on it's lower. The unit Z disk is reflected w.r.t. the curve b_1. The map T_2 behaves similarly.

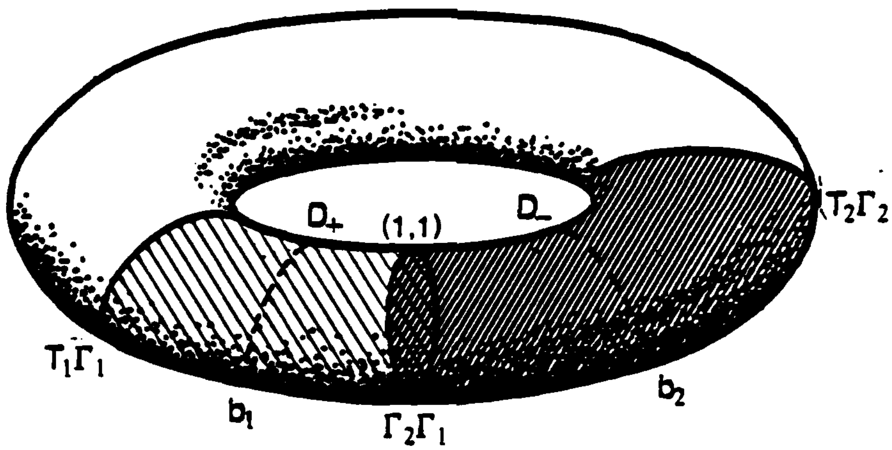

Figure 4.1-3: Representing the domains of analyticity on S

Finally, on the universal covering the regions corresponding to the unit disks are infinite curvilinear strips. The automorphisms T_1 and T_2, correspond to automorphisms of the uniformizing plane. The only automorphisms of the uniformizing plane are linear maps. The condition that $T_i T_i = I$ implies that the linear maps in question are of the form $-u+\Omega_i$, where Ω_i is a constant. The values of the respective constants may be found from the position of the points corresponding to the (z,w) pairs: (1,1), $(1,1/\rho_2)$, $(1/\rho_1,1)$. The computation is relatively easy as we shall later see.

4.1.4 A WIENER-HOPF PROBLEM ON A TORUS

Let us rewrite the boundary equation 4.1-5 on the characteristic curve S. We use the letter P to denote a generic point of S. In the region $\{(z,w): \ |z|,|w|<1\}$ on S, which we denote $D_Z \cap D_W$, permitting ourselves some abuse of notations, the following functional equation holds:

$$(4.1\text{-}8) \qquad A^{10}(P)G^{10}(P)+A^{01}(P)G^{01}(P)+A^{00}(P)G^{00}=0$$

$G^{10}(P)$, $G^{01}(P)$ represent the lifting of the respective functions to the surface S. We choose to abuse our notations rather than complicate them to a point where they are no longer transparent. When a danger of confusion may arise accurate notations will be employed.

In addition to the boundary equation, the unknown functions must be automorphic w.r.t. the respective groups of covering transformations. That is:

$$(4.1\text{-}9) \qquad \begin{aligned} G^{10}(T_1P) &= G^{10}(P) \\ G^{01}(T_2P) &= G^{01}(P) \end{aligned}$$

The problem is to solve 4.1-8 subject to the conditions 4.1-9 and the analyticity of the respective functions in D_Z and D_Z.

Suppose that we solve the equations above for $G^{10}(P)$ and $G^{01}(P)$, then by virtue of 4.1-6 the two functions may be projected onto respective functions of z and w respectively. The projections are analytic in D_Z and D_W respectively. Once the boundary transforms have been determined, it is possible to determine the interior distribution $G^{11}(z,w)$ and the normalizing factor G^{00}. Therefore we should proceed to solve the functional equation described by 4.1-8 and 4.1-9.

The first step is to continue the equations analytically from $D_Z \cap D_W$ to a larger domain. This is easily carried out. Indeed, the coefficients of the equation 4.1-8 are all meromorphic functions over S. The equation 4.1-1 can be immediately continued to $D_Z \cup D_Z$. To continue the functions further we make use of the automorphisms. Applying T_2 to P in the equation 4.1-8 and eliminating $G^{01}(P)$ from the two equations, we get:

$$(4.1\text{-}10) \quad G^{10}(TP)=\alpha(P)G^{10}(P)+\beta(P)$$

where $\alpha(P)$ and $\beta(P)$ are meromorphic coefficients and $T \triangleq T_1 o T_2$.

Let us consider the map T over the torus S. T takes the region $D_Z \cup D_W$ in a spiral movement to an adjacent region (with some overlap). Equation 4.1-10 can be used to extend G^{10} to $T(D_Z \cup D_W)$ and then to the regions generated from $D_Z \cup D_W$ by successive iterations of the operator T. The process may be thought of as pasting function elements over sleeves cut from the torus S. The result is that G^{10} is extended to a function which is meromorphic on a long sleeve which encloses itself in a spiral manner. Perhaps the best description of the surface generated by the pasting procedure, is Escher's ingenious Spirals reproduced in Figure 4.1-4.

In a similar manner we can continue G^{01} to the surface generated by the pasting procedure. Finally, equations 4.1-8 and 4.1-9 should hold on Escher's spiral where the functions G^{10} and G^{01} are meromorphic.

Let us consider the analytic continuation over the uniformizing plane U. The region $D_Z \cup D_Z$ on S is covered by an infinite set of curvilinear strips in the U plane which are parallel to each other w.r.t. the period $2\omega_2$ of S. Let us consider

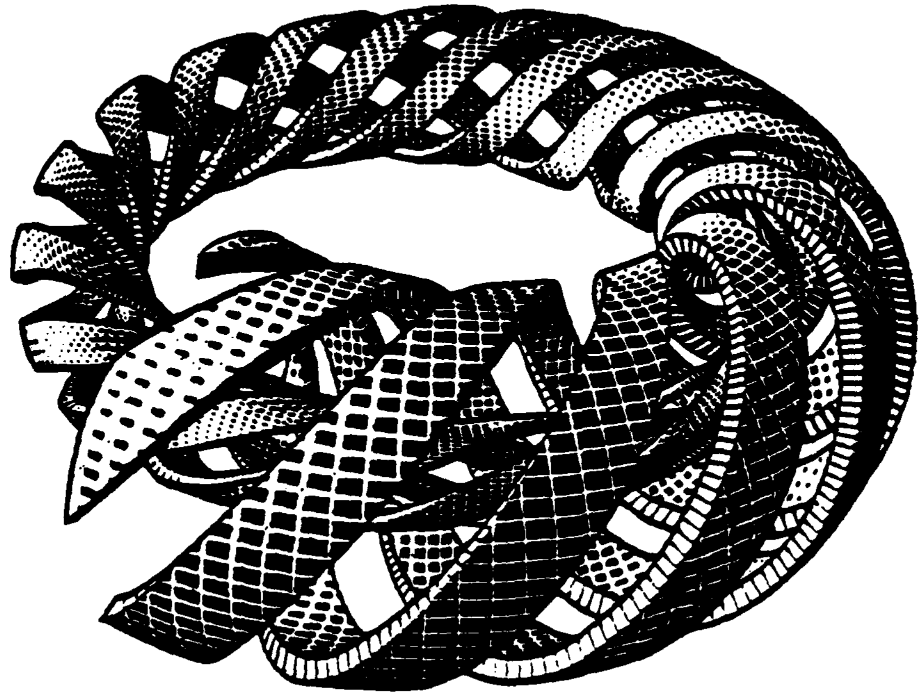

Figure 4.1-4: Escher's spirals representing the Riemann surface of the function $G^{10}(P)$

any one of those strips (i.e., we restrict ourselves to a branch of the cover of U by S and extend that branch from a period parallelogram to a strip by analytic continuation using the periodicity w.r.t. $2\omega_1$).

We start with our particular choice of a strip $D_Z \cup D_Z$ and iterate the operator T to continue the equation and the unknown functions analytically. The map T corresponds to a translation $u \dashrightarrow u+2(\Omega_1-\Omega_2)$ of the U plane. The strips $D_Z \cup D_Z$ are translated in parallel. A translated strip intersects it's preimage. This is merely to say that the strip $D_Z \cup D_Z$ contains a fundamental domain of the map T. When the iterations are carried indefinitely, the whole U plane is covered. Therefore, the function G^{10} may be extended to a meromorphic function over the plane U. Similarly, the function G^{10} may be extended to the whole plane. Finally the principle of permanence of functional relations [SVESH73], implies that equations 4.1-1 and 4.1-9 may be extended to the whole U plane, and that both functions $G^{10}(u)$ and $G^{01}(u)$ are periodic in $2\omega_1$.

4.1.4.1 Summary

Let us summarize our findings, reformulating the problem over the U plane.

1. It is possible to uniformize the characteristic curve with the aid of elliptic functions $z(u)$, $w(u)$ such that $A(z(u),w(u))=0$. Wlog, the two periods of $z(u),w(u)$, i.e., $2\omega_1$ and $2\omega_2$, may be choosen so that the first is real and the second is a purely imaginary number. Furthermore, the origin of the U plane may be selected so that $z(0)=w(0)=1$.

2. The function $z(u)$ is invariant w.r.t the transformation

T_1:u---->-u+$2\Omega_1$, where $z(2\Omega_1)=1$ and $w(2\Omega_1)=1/\rho_2$ (these relations may be used to compute the constant Ω_1). The function w(u) is invariant w.r.t the transformation T_2:u---->-u+$2\Omega_2$, where Ω_2 is defined similarly. The constants Ω_1 and Ω_2 are purely imaginary and have opposite signs, wlog we assume that $Im(\Omega_1)>0$.

3. The two unknown functions $G_+(u) \triangleq G^{10}(z(u))$ and $G_-(u) \triangleq G^{01}(z(u))$ can be extended to the whole U plane where they are periodic w.r.t $2\omega_1$ and satisfy the following relations:

   a. The boundary equation: $G_+(u) = \alpha(u)G_-(u) + \beta(u)$. Here $\alpha(u)$ and $\beta(u)$ are doubly periodic functions.

   b. The authomorphic relations: $G_+(u)$ is invariant w.r.t $T_1(u)$ and $G_-(u)$ is invariant w.r.t. $T_2(u)$.

   c. The analyticity conditions: G_+ is analytic in $D_+ \triangleq D_z$ and G_- is analytic in $D_- \triangleq D_w$.

The problem posed by the third item in the above list is a generalized form of the Wiener-Hopf problem [NOBL53].

In what follows we shall demonstrate how the Wiener-Hopf factorization technique may be generalized to solve problems of the above description. We shall follow the computational details for some of the processes described above, in order to gain insight into the solution process.

4.2 THE TDRW OF GENUS ZERO

4.2.1 THE GEOMETRY OF THE CHARACTERISTIC CURVE

Let us consider a TDRW of genus zero such as the one arising from the isarithmic input control procedure, described in the previous section (see Figure 4.1-1). The characteristic curve is given by

$$(4.2\text{-}1) \qquad 0 = A(z,w) = z^2p(w) + zq(w) + r(w)$$

where $p(w) = \alpha_2$, $q(w) = (1-\alpha_0)w + \alpha_1$ and $r(w) = \alpha_6 w^2 + \alpha_7 w$.

We wish to find a uniformization for the characteristic curve. Namely, it is required to represent z and w as uniform functions of some parameter u so that 4.2-1 is satisfied for each value of u. There are a few methods to obtain uniformization. We choose to employ a method which will work when the genus increases. We uniformize the characteristic curve in terms of integrals of the first kind on the curve.

The idea is simple. Equation 4.2-1 may be thought of as an integral of

$$(4.2\text{-}2) \qquad 0 = dA(z(u),w(u)) = \frac{\partial A}{\partial z}dz + \frac{\partial A}{\partial w}dw$$

along the characteristic curve.

Now a simple computation shows that

$$(4.2\text{-}3) \qquad \frac{\partial A}{\partial z} = 2zp(w) + q(w) = \pm[\Delta_2(w)]^{1/2}$$

where $\Delta_2(w) \triangleq [q(w)]^2 - 4p(w)r(w)$ is the discriminant of the equation.

Similarly, one may compute $\frac{\partial A}{\partial w}$. Replacing the corresponding expressions in

4.2-2 we get an Euler equation for the characteristic curve [HALP88, HANC58, SIEG69]. Namely

(4.2-4) $$0 = dz/[\Delta_1(z)]^{1/2} \pm dw/[\Delta_2(w)]^{1/2}$$

(This equation was employed by Euler to obtain addition theorems for elliptic functions. Euler's investigations mark the beginning of the theory of elliptic functions. Further information about the fascinating history of this theory may be found in any of the above references.)

We shall proceed to integrate 4.2-4 in terms of trigonometric functions when the genus of the walk is 0 and elliptic functions when the genus is 1. We follow Euler's method verbatim. Indeed our uniformization process is merely that of seeking a curve over which the characteristic equation is an addition theorem.

Let us consider the discriminant $\Delta_1(z)$ in details. When the genus of the RW is 0 the function $\Delta_1(z)$ is a quadratic polynomial

(4.2-5) $$\Delta_1(z) = a_1z^2 + 2b_1z + c_1 = -(\delta_1{}^2/a_1)\left[\,1 - (a_1z+b_1)^2/\delta_1\,\right]$$

where

$$a_1 \triangleq \alpha_0{}^2 - 4\alpha_2\alpha_6$$

$$b_1 \triangleq \alpha_7\alpha_0 - 2\alpha_1\alpha_5$$

$$c_1 \triangleq \alpha_7{}^2 - 4\alpha_6\alpha_8$$

and $\delta_1 \triangleq \left[\, b_1{}^2 - a_1c_1 \,\right]^{1/2}$ may be checked to be real*. The function $\Delta_2(w)$

*We choose the positive root.

may be computed in a similar manner.

Now consider the following transformation of the variables:

$$x \triangleq (a_1 z + b_1)/\delta_1, \qquad y \triangleq (a_2 w + b_2)/\delta_2 .$$

Clearly $dx = (a_1/\delta_2)\,dz$ and $dy = (a_2/\delta_2)\,dw$.

In terms of these new variables it is possible to rewrite 4.2-4 as*

$$(4.2\text{-}6) \qquad 0 = dx/(1-x^2)^{1/2} \pm dy/(1-y^2)^{1/2}$$

If u is defined (up to a constant of integration) by:

$$du \triangleq dx/(1-x^2)^{1/2}$$

then, equation 4.2-6 may be integrated to give:

$$(4.2\text{-}7) \qquad \begin{aligned} x(u) &= \mathrm{Cos}(u - \Omega_1) \\ y(u) &= \mathrm{Cos}(u - \Omega_2) \end{aligned}$$

Here Ω_1 and Ω_2 are arbitrary constants of integration. This equation implies the following representation of z and w in terms of u:

$$(4.2\text{-}8) \qquad \begin{aligned} z(u) &= (1/a_1)\left[\,\delta_1 \mathrm{Cos}(u - \Omega_1) - b_1\right] \\ w(u) &= (1/a_2)\left[\,\delta_2 \mathrm{Cos}(u - \Omega_2) - b_2\right] \end{aligned}$$

---

*Note that $a_1 = a_2$, so both factor out from the equation.

This last equation is, up to a selection of integration constants, a uniformization of the characteristic curve in terms of the uniformizing parameter u.

It is enough to fix one point of the curve in order to determine the constants of integration. Let us choose the origin of the uniformizing plane so that $z(0) = w(0) = 1$.

A simple computation gives the following values of Ω_1 and Ω_2.

(4.2-9) $\quad \mathrm{Cos}\Omega_1 = (a_1 + b_1)/\delta_1$ and $\mathrm{Cos}\Omega_2 = (a_2 + b_2)/\delta_2$

Let us note in passing that the cover automorphisms of the characteristic curve are given by:

(4.2-10)
$$T_1(u) = 2\Omega_1 - u$$
$$T_2(u) = 2\Omega_2 - u$$

Namely,

(4.2-11) $\quad z(T_1(u)) = z(u)$ and $w(T_2(u)) = w(u)$

Finally, the composition of the two automorphisms yields:

(4.2-12) $\quad T(u) \triangleq (T_1 o T_2)(u) = u + 4\Omega$

Where $\Omega \triangleq (1/2)\left[\Omega_1 - \Omega_2\right]$. The operator $T_2 o T_1$ is precisely T^{-1}.

To conclude this section, it is possible to obtain a uniformization of the characteristic curve in terms of trigonometric functions, using integrals of the

first kind over the characteristic curve. The uniformization is given in equation 4.2-8, where the integration constants are provided by 4.2-9.

In what follows we solve the boundary equation by uniformizing it over the u-plane. We shall consider first the representation of the domains of analyticity of the two unknown transformed boundary distributions. Then we apply a Wiener-Hopf technique to derive the solution.

4.2.2 THE DOMAINS OF ANALYTICITY

The domains of analyticity of the transformed boundary distributions $G^{10}(z)$ and $G^{01}(w)$, i.e., $D_z(1)$ and $D_w(1)$, are conformally represented upon the u plane, by the inverse of the uniformizing map 4.2-8. Let D_+ and D_- represent the corresponding regions in the u-plane, respectively.

It is easy to check that $a_i + 2b_i + c_i > 0$ so that $(a_i + b_i)^2 > \delta_i^2$. This, in turn, implies that $(a_i + b_i)/\delta_i > 1$. Therefore the definition of Ω_i (4.2-9) implies that both constants Ω_i are *purely imaginary*.

Now, $w(0)$ and $w(2\Omega_1)$ are the two points on the characteristic curve, which correspond to $z = 1$. Since $w(0)=1$, we conclude that $w(2\Omega_1)=1/\rho_2$. Therefore if $\rho_2 < 1$, then the point $u = 2\Omega_1$ cannot belong to D_-. In a similar way one can show that $\rho_1 < 1$ implies that $u = 2\Omega_2$ cannot belong to D_+. Therefore, the assumption of strong stability (i.e., $\rho_i<1$) implies that *the two constants* $\mathrm{Im}(\Omega_i)$* *cannot have the same sign*. Therefore, without loss of generality, $\mathrm{Im}(\Omega_1) > 0$ and $\mathrm{Im}(\Omega_2) < 0$.

The domains of analyticity D_+ and D_- are depicted in Figure 4.2-1. Note that D_+ (D_-) is symmetric w.r.t the line $\mathrm{Im}(u) = \mathrm{Im}(\Omega_1)$ ($\mathrm{Im}(u) = \mathrm{Im}(\Omega_2)$).

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*Here Im(u) denotes the imaginary part of the number u.

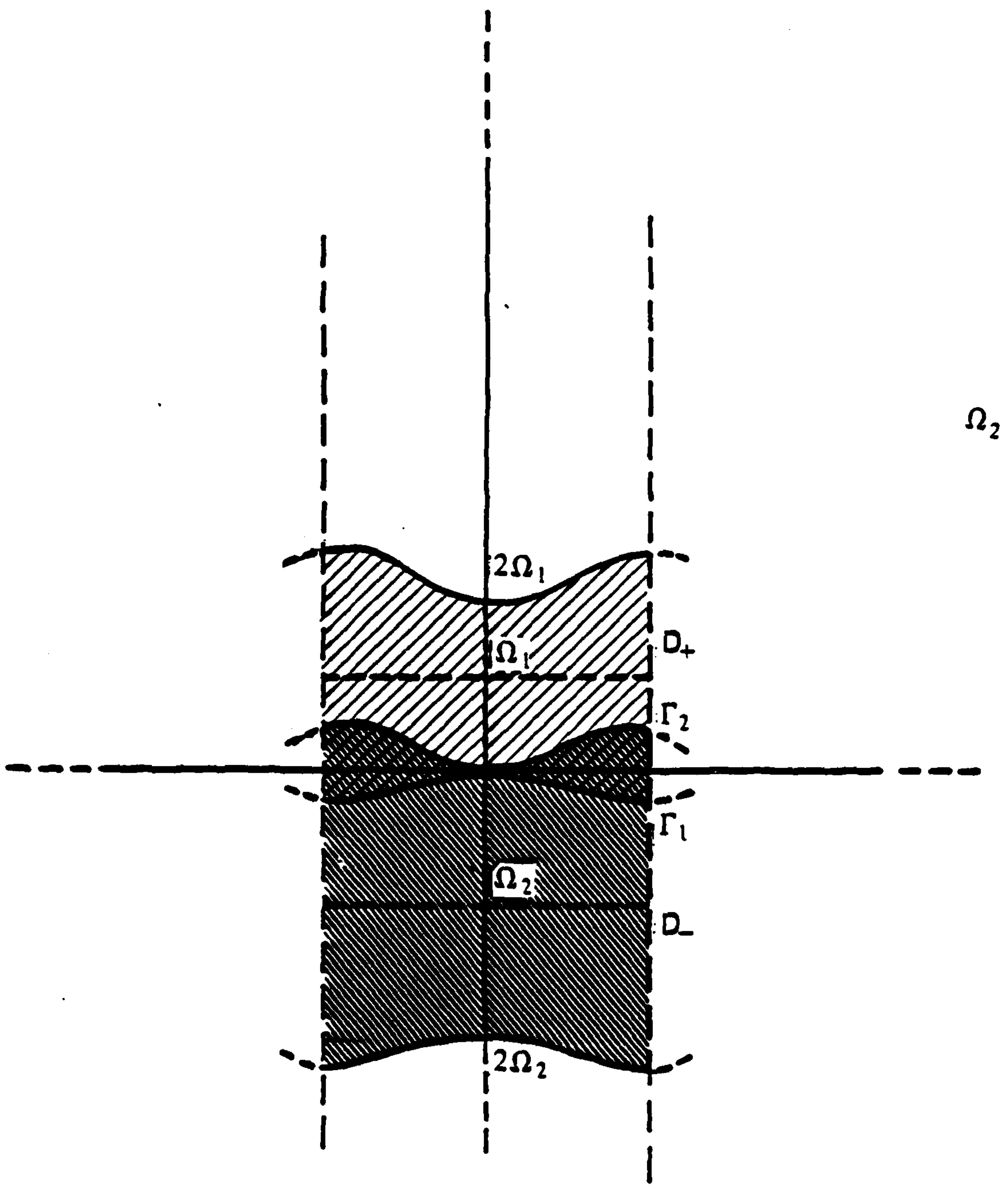

Figure 4.2-1: Domains of analyticity for the TDRW of genus 0

4.2.3 THE BOUNDARY EQUATION

Let us consider the representation of the boundary equation over the uniformizing plane. Define $G_+(u) \triangleq G^{10}(z(u))$ and $G_-(u) \triangleq G^{01}(w(u))$. Over the uniformizing plane the boundary equation assumes the form:

(4.2-13) $\quad 0 = A^{10}(u)G_+(u) + A^{01}(u)G_-(u) + A^{00}(u)$

Here $A^{10}(u) \triangleq A^{10}(z(u),w(u))$ and $A^{01}(u)$ and $A^{00}(u)$ are defined similarly.

The analyticity conditions are:

(4.2-14) $G_+(u)$ and $G_-(u)$ are analytic and zero free in D_+ and D_- respectively.

Finally, the two unknowns are invariant w.r.t the autorphisms T_1 and T_2 respectively:

$$G_+(2\Omega_1 - u) = G_+(u)$$

(4.2-15)

$$G_-(2\Omega_2 - u) = G_-(u)$$

Let us restrict ourselves to the case of *symmetric* characteristic curve, i.e., $A(z,w) = A(w,z)$. In this case the computations are simplified and the solution process can be better understood. The generalization to the non-symmetric case is straight-forward but requires somewhat cumbersome computations of no interest.

In the case of symmetric characteristic curve $\Omega_1 = -\Omega_2 = \Omega$ (wlog we assume that $Im(\Omega) > 0$).

4.2.3.1 The associated homogeneous equation

Consider the homogeneous equation associated with the boundary equation 4.2-13:

(4.2-16) $X_+(u) = R(u)X_-(u)$

where $R(u) \triangleq - A^{10}(u)/A^{01}(u)$. Wlog it is possible to assume that the strip of common analyticity $D_- \cap D_+$ does not contain any zeros or poles of R(u) nor their symmetric points w.r.t the line $Im(z)=\pm 2\Omega$. Let us assume that X_+ and X_- are a solution pair satisfying the respective invariance and analyticity conditions. Suppose we can represent R(u) as a ratio $\Psi_-(u)/\Psi_+(u)$, where Ψ_+ (Ψ_-) is analytic and zero free function in the domain D_+ (D_-), invariant w.r.t T_1 (T_2). Equation 4.2-16 can be rewritten as $X_+\Psi_+=X_-\Psi_-$. The function f(u) described by the two sides of the last equality is analytic in $D_+ \cup D_-$ and is invariant w.r.t both transformations T_1 and T_2, and thus w.r.t their composition T. Therefore f(u) is a doubly periodic function analytic in the period parallelogram; this can only occur iff f(u) is a constant. Finally, if a factorization as the one described above, is possible then $X_+(u)=C/\Psi_+(u)$ where C is some constant ($X_-(u)$ may be computed similarly).

In conclusion we see that the homogeneous problem may be solved if we can find a factorization of the coefficient R(u), as suggested above. It is possible to pursue a general approach to the factorization problem, resulting in integral expression of the factors. However, in our case the coefficient R(u) is a rational trigonometric function and the factorization can be greatly simplified to a purely algebraic process (this simplification resembels the classical Wiener-Hopf factorization problem for rational coefficients).

4.2.3.2 Factorization of the homogeneous equation

The function $R(u)$ is obtained through a rational combination of simply periodic functions and therefore can be represented in the form [HANC58]:

$$(4.2\text{-}17) \qquad R(u) = C\exp(2kiu)\,\frac{\prod_{i=1}^{n}\left[1 - \exp[2i(u-a_i)]\right]}{\prod_{j=1}^{m}\left[1 - \exp[2j(u-b_j)]\right]}$$

where a_i and b_j are the respective finite zeros and poles of $R(u)$, C is a constant and k is an integer.

Therefore, it is sufficient to solve the simplified homogeneous equation for coefficients of the form $[1 - \exp(u-c)]$, $1/[1 - \exp(u-c)]$ and $\exp(iku)$. Let us proceed to obtain these solutions.

4.2.3.3 Solution of the simplified homogeneous problem

Consider the following simplified factorization problem:

(4.2-18) $\Psi_+(u) = \alpha(u)\,\Psi_-(u)$

Where $\alpha(u)$ is either $[1 - e^{i(u-c)}]$, $1/[1 - e^{i(u-c)}]$, or e^{iku}. The functions $\Psi_\pm(u)$ are analytic and zero free in the respective domains and satisfy the authomorphic relations:

(4.2-19) $\Psi_+(2W - u) = \Psi_+(u)$ and $\Psi_-(-2W - u) = \Psi_-(u)$

Consider the first type of coefficient, namely: $\alpha(u) = [1 - e^{i(u-c)}]$. Let us assume first that $c \in D_-$ and consider the following functions:

(4.2-20) $\phi_1(u) = \prod_{n=0}^{\infty} [1 - e^{i(u-c+4n\Omega)}]$

$$\phi_2(u) = \prod_{n=0}^{\infty} [1 - e^{i(-u-c+2\Omega+4n\Omega)}]$$

The inequality $\mathrm{Im}(\Omega) > 0$ shows that $e^{4i\Omega} < 1$, implying the absolute convergence of the infinite series $\sum_{n=0}^{\infty} e^{(i4n\Omega)}$; this, in turn ([HANC58] chapter I), implies that the infinite products above converge absolutely to entire functions.

Clearly, $\phi_1(u+4\Omega) = \phi(u)/(1 - e^{i(u-c)})$ and $\phi_2(u+4\Omega) = \phi(u)(1 - e^{i(-u-c-2\Omega)})$. Define $\Psi_+(u) \triangleq \phi_1(u)\phi_2(u)$ and $\Psi_-(u) \triangleq \Psi_+(u)/[1 - e^{i(u-c)}]$ then:

1. $\Psi_+(u) = (1 - e^{i(u-c)})\,\Psi_-(u)$ by definition of Ψ_-.

2. $\Psi_+(2\Omega-u) = \Psi_+(u)$ since $\phi_2 = \phi_1(2\Omega-u)$.

3. $\Psi_-(-2\Omega-u) = \Psi_+(-2\Omega-u)/[1 - e^{-i(u+c+2\Omega)}] =$

$= \Psi_+(u+4\Omega)/[1 - e^{-i(u+c+2\Omega)}] = \Psi_+(u)/[1 - e^{i(u-c)}] = \Psi_-(u)$

4. $\Psi_+(u)$ has no poles and its only zero in $D_+ \cup D_-$ is c (note that 2Ω-c falls outside the critical region).

5. $\Psi_-(u)$ has no poles and no zeros in $D_+ \cup D_-$.

Therefore the pair $\Psi_+(u)$ and $\Psi_-(u)$ solve the factorization problem for the coefficient $[1 - e^{i(u-c)}]$ for $c \in D_-$.

In a similar way it is possible to show that the functions $\Psi_+(u) \triangleq 1/\phi_1(u)\phi_2(u)$ and $\Psi_-(u) \triangleq \Psi_+(u)[1 - e^{i(u-c)}]$ solve the factorization problem for the coefficient $[1 - e^{i(u-c)}]^{-1}$, when $c \in D_-$.

Let us turn now to the case $c \in D_+$. It is possible to assume, wlog, that the factors having roots in D_+ appear with arguments having a negative sign, i.e., have the form $[1 - e^{-i(u-c)}]$.* Consider the functions:

$$\phi_3(u) = \prod_{n=0}^{\infty} [1 - e^{i(-u+c+4n\Omega)}] \tag{4.2-21}$$

$$\phi_4(u) = \prod_{n=0}^{\infty} [1 - e^{i(u+c-2\Omega+4n\Omega)}]$$

Define $\Psi_-(u) \triangleq \phi_3(u)\phi_4(u)$ and $\Psi_+(u) \triangleq \Psi_-(u)[1 - e^{-i(u-c)}]$ then the functions Ψ_- and Ψ_+ may be checked to solve the factorization problem for $[1 - e^{-i(u-c)}]^{-1}$ for $c \in D_+$. The factorization problem for $[1 - e^{-i(u-c)}]$ may be solved in a similar way.

Suppose now that $c \notin D_+ \cup D_-$. If Im(c)>0, then inspection shows that the

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*Simply divide and multiply the original coefficient by the respective exponent.

factorization problem is solved by the same functions as if $c \in D_+$. Similarly if $Im(c)<0$ the solution is provided by the same process as if $c \in D_-$.

Finally, let us turn to the simplified homogeneous problem for coefficients of the form $\alpha(u) = e^{iku}$. We claim that if $k \neq 0$ then the factorization problem does not possess a solution. In fact, suppose Ψ_+ and Ψ_- solve the factorization problem for e^{iu} (clearly it is sufficient to consider k=1 only). Then the functions Ψ_+ and Ψ_- are doubly periodic of the third kind ([HANC58], Ch. V). Moreover, they do not possess any zeros or poles in the fundamental parallelogram. The only such functions are constants (ibid.).

To conclude the discussion, we have developed *closed form solution of the simplified homogeneous equation, for the three cases of coefficients. The solution of the general homogeneous problem can be expressed as a product of solutions of simplified problems. In the case of exponential coefficients we have shown that a necessary and sufficient condition for solvability of the homogeneous problem is that the exponential coefficient disappears.*

4.2.3.4 Solution of the non-homogeneous problem

The solution of the non-homogeneous boundary problem can be carried out in a manner similar to the usual Wiener-Hopf technique for non-homogeneous problems. Consider again equation 4.2-13. Let us assume that X_+ and X_- are the solution pair of the respective homogeneous equation for the coefficient $R(u) \triangleq -A^{10}(u)/A^{01}$. Let $Y_+(u) \triangleq X_+(u)G_+(u)$ and $Y_-(u)$ be defined similarly. The functions Y_+ and Y_- satisfy following variant of 4.2-13:

$$(4.2\text{-}22) \quad Y_+(u) = Y_-(u) + \beta(u)$$

Where $\beta(u) \triangleq X_-(u)A^{00}(u)/A^{01}(u)$

In additon, the functions Y_+, Y_- satisfy the respective automorphic and the analyticity conditions.

Suppose we could represent the function $\beta(u)$ as : $\beta(u) = \beta_-(u) - \beta_+(u)$, where the two tems in the decomposition satisfy the respective analyticity and authomorphic conditions, then the functions $Y_+ + \beta_+ = Y_- - \beta_-$ represent analytic continuation of each other into the fundamental parallelogram and are analytic there and doubly periodic, therefore constants. Therefore, $G_+(u) = \beta_+(u)/X_+(u)$ and $G_-(u)$ can be expressed similarly. Thus, we need to solve the decomposition problem for the coefficient $\beta(u)$.

Again, it is possible to solve this last decomposition problem in terms of integrals, using methods for solving the Riemann Problem with automorphisms. However, a better approach is to use the rationality of the function A^{00}/A^{01} and the theta-function structure of X_-. The idea is to use the well known representation of doubly-periodic functions in terms of a "partial fraction

expansion" and then solve the above equation for each term in the expansion. The details of the computation can be found [HANC58] and would be a simple yet cumbersome repetition of the Wiener-Hopf solution technique for non-homogeneous problems with rational coefficients [NOBL53]. This, we feel, would carry us beyond the scope of this dissertation and is thus avoided here.

4.2.3.5 Conclusions

In the previous section we have illustrated the solution process for a quite general TDRW of zero genus. As we have proceeded through the stages of the solution process, we have watched the complexity of the solution growing beyond actual applicability. We can conclude that, computationally speaking, the solution of the general TDRW in terms of "closed form formulas" is most probably unfeasible.

Therefore, the above process should not be considered as a recommended procedure for solving the problem (simulation and approximate numerical solutions are much simpler). The major value of the process lies in the possibility of deriving suitable approximations for the solution of TDRW from its geometric structure.

For example, one can approximate the behavior of $G_+(u)$, $G_-(u)$ in terms of their dominating poles. The theta-like functions appearing in the denominators of the functions have one "nice" property, they converge (as infinite products) very rapidly. Thus, the behavior of the solutions is completely dominated by their most signficant poles.

5. THE ECOLOGY OF RANDOMIZED CHANNEL SHARING SCHEMES

5.1 CAPACITY OF RANDOMIZED ACCESS SCHEMES

5.1.1 THE PROBLEM OF CAPACITY

In what follows, we consider a PRNET which uses a coin tossing, Slotted-ALOHA access scheme to share the communication channel. The network model is the one described in section 1.1. Let us assume that all traffic is directed towards a single destination. The parameters of interest are the input distribution $V \triangleq (v_1, v_2, \ldots, v_N)$, v_i being the fraction of the input traffic arriving at PR_i; the overall input rate γ, i.e., the the expected number of packets arriving at the PRNET per slot; the routing policy $R = (R_{ij})$, R_{ij} being the fraction of traffic routed from PR_i to PR_j; the throughput distribution $U \triangleq (u_1, u_2, \ldots, u_N)$, U_i being the fraction of the total throughput processed by PR_i; the total "throughput" s, i.e., the overall expected number of packets which are successfully transmitted at each slot; the transmission policy $P = (p_1, p_2, \ldots, p_N)$, p_i being the transmission probability of PR_i. The queueing process at each of the buffers $Q \triangleq (Q_1, Q_2, \ldots, Q_N)$ describes a multidimensional, positive, integer random walk.

Let us fix the routing R and the input distribution V. As we raise the input rate γ some network queues are driven into instability. As long as all queues are stable, the input distribution V and the routing R determine the throughput distribution U [KLEI64]. The total input rate determines the total throughput rate, as expressed by the equation $s = \gamma \times n$ where n is the expected

communication path length. The number n is completely determined by the routing R and the input distribution V. Therefore, given R and V, the input rate γ determines the total throughput and vice versa. Therefore, let us consider the total throughput of the network s. A suitable notion of network *capacity* along U should be defined as

$$C(U) \triangleq SUP\{ s \mid S \triangleq s \times U \text{ is an attainable throughput vector}\}$$

This requires that we define the meaning of "attainable" precisely. A throughput s is *attainable*, if a transmission policy P exists for which the random walk Q, determined by the pair (S,P), is stable. That is, none of the packet queues wanders to infinity. Thus the problem of capacity is intimately linked to the stability problem of our queueing random walk.

Ideally we would like to examine all pairs (S,P) - U being fixed - which define a stable system and find the L.U.B of all the attainable throughputs - s. Unfortunately, such a scheme is computationally infeasible since we do not know a characterization of the stable (S,P)'s. As a matter of fact, only little is known about stability of a multidimensional, bounded random walk.

In the case of one PRU, only one road to instability exists. All the PRU has to do in order to become unstable is to let his queue drift towards infinity (on the average). The case with the multi- queue problem is different: there are many styles which the PRNET may choose to turn unstable.

In the absence of a computational characterization of the capacity notion defined above, alternative notions should be explored. One method of attack upon the capacity problem is to assume some "reasonable" asymptotic behavior

of the network as it is driven into the instability region. Such an approach proved successful for one- hop networks. In this case the natural assumption is that instability is approached in heavy-traffic; i.e., all queues become busy all the time.

The assumption of heavy-traffic yields a "worst case" notion of capacity. Loosely speaking, the heavy-traffic capacity represents the best performance that the PRNET may obtain under the worst traffic conditions that may occur. It is the maximum "payoff" that we can derive from the PRNET when the users are placing the worst combination of demands. Therefore, borrowing from the terminology of game theory, we wish to consider a max-min notion of capacity.

The max-min capacity of the PRNET is defined as follows. Assume that all queues are busy; then, given a routing matrix R, for each transmission policy P it is possible to compute the throughput vector $S = \underline{S}(P)$. The map $S = \underline{S}(P)$ transforms the set of all transmission policies into a compact domain representing all heavy-traffic-attainable throughputs. Consider the ray in the S space in the U direction. The maximal value of heavy-traffic- attainable S along the ray $S = s \times U$ is called the "heavy-traffic" capacity along U.

Fortunately (or unfortunately, depending on your point of view) the heavy-traffic assumption (namely, that all queues are busy when the network is driven into instability) does not necessarily describe the actual behavior of some classess of policies in general multi-hop networks. That is, the max-min capacity may be substantially lower than the actual capacity. The reason for the disparity, as we shall see, is the ability of some PRNETS to select a transmission policy which will take advantage of the hearing topology and the

input distribution. Such policies phase the transmissions in a manner which eliminates wasteful collisions and a heavy-traffic condition altogether.

Nevertheless, we shall adhere to the concept of max-min capacity and consider networks whose actual capacity is different to be singular. There are two advantages to this approach. First, the problem of computing the max-min capacity is purely combinatorial and reasonably tractable, while the problem of computing the actual capacity is untractable. Second, the ability of an ALOHA policy to phase transmissions is indeed a singular phenomenon which depends completely upon some peculiar properties of the hearing topology and the input distribution; most networks do not possess this peculiar property and may have a hard time detecting its existence once it does exist.

Our considerations will be both descriptive and prescriptive. Indeed, the problem of capacity is intimately linked with the problem of choosing an optimal transmission policy, that is, a policy that attains the capacity. This is not unlike the Information theoretic definition of channel capacity.

We have had more success with the characterization of optimal policies of transmission. Indeed, we shall employ some simple ideas of mathematical economics to derive necessary conditions for optimality. This characterization lends itself to a very simple distributed implementation.

In what follows, we shall consider first a one hop PRNET. We reconsider the analysis of such a network due to N. Abramson in order to develop an insight to the ideas employed later. We proceed to solve the (max-min) capacity problem for a tandem network. The problem of singularity is examined next. Finally,

we characterize optimal transmission policies and describe a quasi-static distributed algorithm to implement those policies.

5.1.2 ONE HOP PRNETS

This section summarizes some of the capacity results for one- hop PRNETs [ABRA73, LAM74, KLEI76]. The purpose of this review is to serve as an introduction to some of the concepts which we develop later.

Let us consider a one-hop PRNET and fix the input distribution V (note V = U). We wish to determine the capacity C(U) of the network.

In solving the capacity problem Abramson assumes implicitly that when the capacity limit is approached, all queues in the network become busy. Under this "heavy-traffic assumption" the expression for the throughput S in terms of the transmission policy P is given by:

(5.1-1) $S_i = [p_i/(1-p_i)] \times E(P)$

where

(5.1-2) $E(P) \triangleq \prod_{j=1}^{N} (1-p_j)$

We shall use the name: *Abramson operator* to designate the operation

(5.1-3) $S \triangleq \underline{S}(P)$

which is defined by equation 5.1-1.

The operator $\underline{S}$ transforms the domain of transmission policies $\Lambda \triangleq [0,1]^N$, onto the domain of heavy-traffic-attainable throughputs. Figure 5.1-1 describes the operation of Abramson operator for the case N = 2. We see that the domain of attainable throughputs forms a curvilinear simplex. We shall be interested in the upper boundary surface of the throughput domain. This

surface represents the throughput vectors S which are not dominated by any other throughput vector.

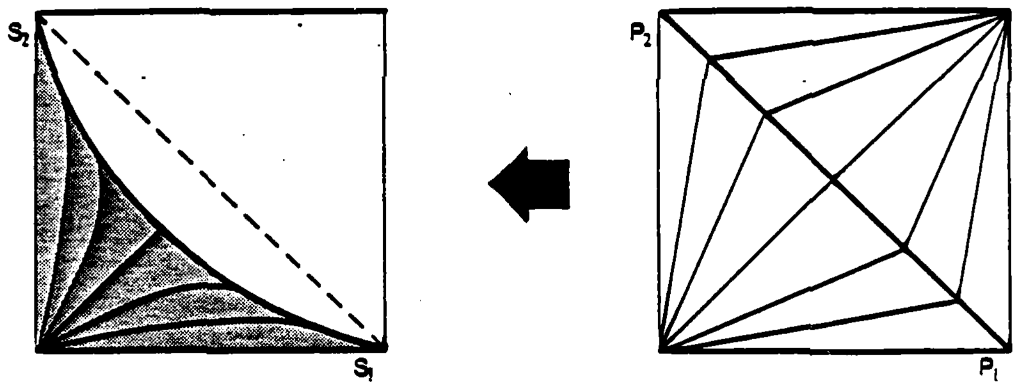

Figure 5.1-1: Abramson's heavy-traffic throughput operator

A throughput vector S is called *Pareto optimal* iff

1. *it is heavy-traffic attainable, i.e., $S = \underline{S}(P)$ for some transmission* policy P.

2. it is not dominated by any other throughput, i.e., there exists no attainable S', such that S'>S. (here S'>S means that for all $1 \leq i \leq N$, $S'_i \leq S_i$ with at least one strict inequality)

A policy P which attains a Pareto-optimal throughput is called a *Pareto-optimal policy*. The idea of using Pareto optimality as our notion of optimality is simple. The Pareto optimal policies represent a reasonable choice of a decentralized resource-sharing network policy; for it is impossible to improve the throughput of all network members simultaneously. A user who wishes to obtain a larger share of the channel, can only do so at the expense of other users.

The Pareto optimal policies may be characterized as the points at which the Jacobian of the Abramson operator becomes zero.

The above statement has a simple geometric proof. Indeed, consider a Pareto-optimal policy P^0 and let S^0 be the Pareto-optimal throughput corresponding to P^0. A small perturbation of P^0 leads to a small perturbation of S^0. The respective perturbations are related through

(5.1-4) $\Delta S = \partial \underline{S}(P^0) \times \Delta P$

where $\partial \underline{S}$ is the Jacobian matrix of $\underline{S}(P)$.

If P^0 is an internal point of $\Lambda = [0,1]^N$, it admits a set of perturbations containing a neighborhood of zero. The extremality of S^0 implies that the admissible perturbations of S^0 must not contain a neighborhood of zero. This is possible iff the Jacobian matrix is singular. Thus, the Jacobian determinant of the network must be zero.

A simple computation [ABRA73] shows that a necessary and sufficient condition for a policy to be Pareto optimal is that:

(5.1-5) $1 = p_1 + p_2 + \ldots\ldots\ldots + p_N$

The capacity problem has a straightforward solution. Indeed, the input distribution U constrains the attainable throughputs to the ray through the origin at the direction U. The capacity along U is the value of s for which the ray $S = s \times U$ crosses the surface of Pareto optimal throughputs. This is demonstrated in Figure 5.1-2.

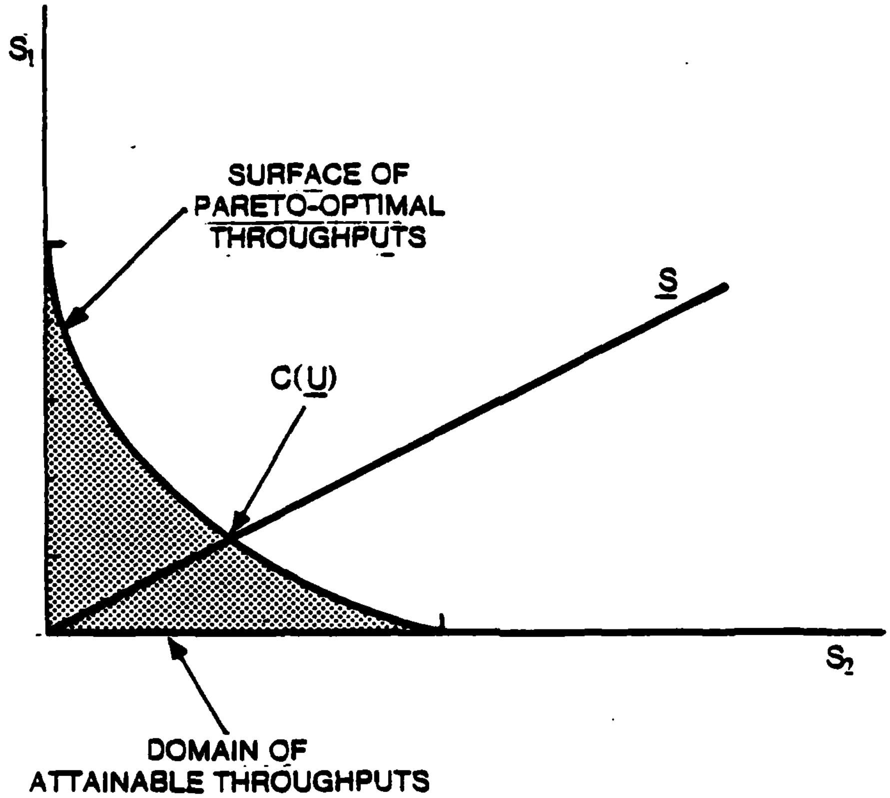

Figure 5.1-2: Geometry of the heavy-traffic capacity problem

When the input distribution is uniform, i.e., $U = (1,1,........,1)$ the capacity assumes its minimal value:

(5.1-6) $C_N = (1 - 1/N)^N$

which converges to $1/e$ when N grows to infinity.

The above derivation rests upon the heavy-traffic assumption. In a one hop network it seems reasonable to assume that this max-min capacity coincides with the actual capacity. The notion of Pareto optimality is the natural geometric formulation of the capacity problem.

5.2 CAPACITY OF A TANDEM

5.2.1 WHY TANDEMS ?

A tandem is the simplest multi-hop carrier of packet radio traffic; its investigation will provide some principles of capacity analysis and operation for multi-hop networks.

We shall consider a tandem of PRUs using a coin-tossing Slotted-ALOHA access scheme to share the communication resource. Each PRU hears his two neighbors only, as depicted in Figure 5.2-1.

Packets arrive from a Bernoulli source to the "upper" end of the tandem. The PRUs serve as repeaters which forward the packets down the tandem to the station at the "lower" end.

Let us name the PRUs according to their position in the tandem: PR_i $1 \leq i \leq N$, where N is the total length of the tandem.

Clearly the input distribution is $V \triangleq (0,0,\ldots\ldots,1)$ while the throughput distribution is $U = (1/N)(1,1,\ldots\ldots,1)$. The average length of a communication path is N, thus $s = \gamma \times N$. We shall abuse the name "throughput" and notation S to denote the input rate γ as well as the throughput of each PRU. Thus S here corresponds to 1/N-th of the throughput in the sense of the previous section.

First we search over wide subclasses of reasonable transmission policies for the actual capacity. We shall see that those policies which we consider for candidates (to obtain the capacity) all yield a single notion of capacity identical to the heavy-traffic capacity. We then resort to this heavy-traffic capacity and

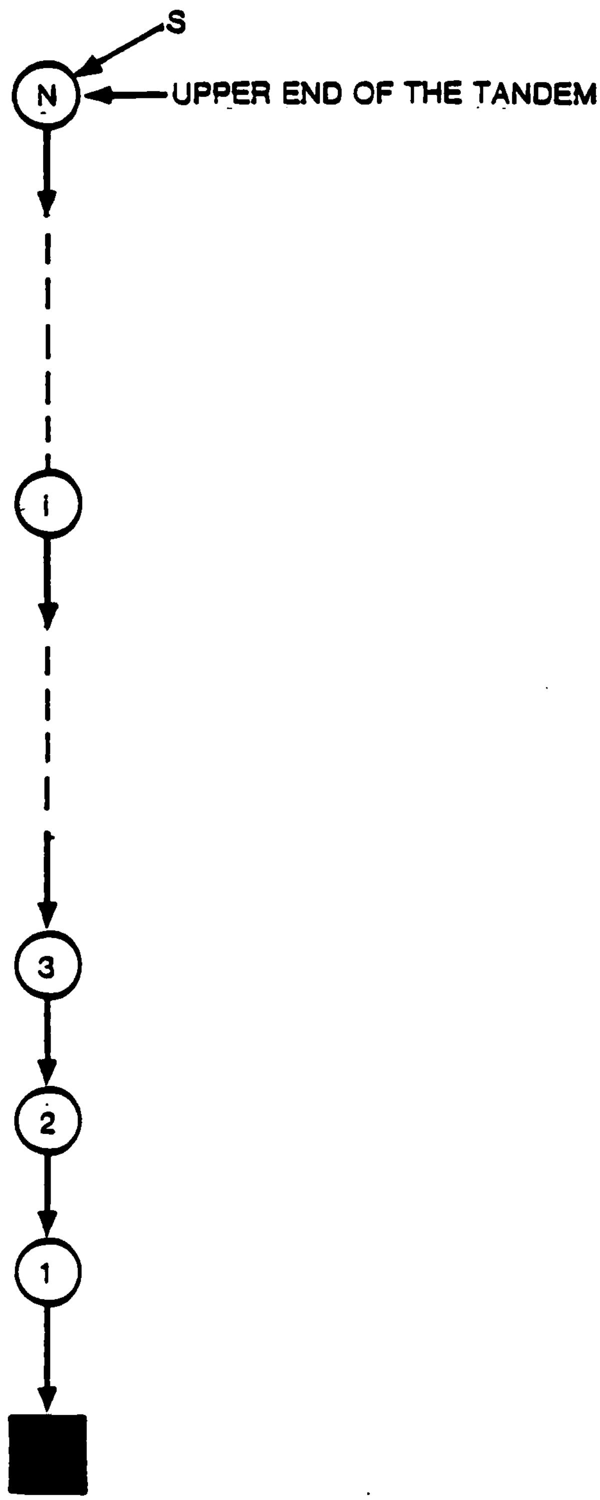

Figure 5.2-1: A tandem network

compute its value as well as its asymptotic behavior for very long tandems. Let us proceed to define policy classes of interest.

One set of policies which the tandem may choose is the set of so-called *polite policies*. Under a polite policy P, each member of the tandem, PR_i, guarantees that even when the whole tandem is busy, the rate of service at PR_i is faster then the rate of arrivals to PR_i.

Formally, a policy P is *polite* if:

(5.2-1) $\quad PS(N) < PS(N-1) < \ldots\ldots\ldots\ldots < PS(i) < \ldots\ldots\ldots < PS(1)$

where

$PS(i) \triangleq p_i(1-p_{i-1})(1-p_{i-2})$

A polite policy attains any throughput S, such that S<PS(N). This intuitive triviality has a straightforward uninteresting proof which we omit.

When we try to blow up the tandem in a polite way, all queues along the tandem will grow simultaneously. Thus we have our first style to become unstable, and with it comes the notion of *polite capacity*:

(5.2-2) $\quad C_N \triangleq SUP\{S \mid \text{There exists a polite policy P which attains S}\}$

Another family of policies which guarantee stability is the family of *fair* policies. A policy is *fair* if:

(5.2-3) $\quad PS(N) = PS(N-1) = \ldots\ldots\ldots = PS(i) = \ldots\ldots\ldots = PS(1)$

A fair policy is stable for any throughput S<PS(N). Again, if we blow the

tandem in a fair way, all queues grow simultaneously. We let C_N^* denote the *fair capacity*. Note that the class of fair policies generates a capacity notion identical to the heavy-traffic capacity.

An almost natural result is that the two notions of capacity which we introduced, yield the same capacity. Indeed:

LEMMA

1. *There is a fair policy* P^* *which attains* C_N^*, *that is:*

$$C_N^* = p_N^*(1-p_{N-1}^*)(1-p_{N-2}^*) = \ldots = p_i^*(1-p_{i-1}^*)(1-p_{i-2}^*) = \ldots = p_1^*$$

2. *The fair and polite capacities are the same.*

PROOF:

(1) follows directly from the definition. To prove (2) we show first: $C_N \geq C_N^*$.

Let $\epsilon > 0$ be arbitrarily small and $S \triangleq C_N^* - \epsilon$, then:

$$p_1^* = p_2^*(1-p_1^*) = \ldots = p_{N-1}^*(1-p_{N-2}^*)(1-p_{N-3}^*) = p_N^*(1-p_{N-1}^*)(1-p_{N-2}^*) > S$$

here P^* is the policy of (1). Now decrease p^*_N by a small amount to get a p_N so that the rightmost inequality is preserved. We get:

$$p_1^* = p_2^*(1-p_1^*) = \ldots = p_{N-1}^*(1-p_{N-2}^*)(1-p_{N-3}^*) > p_N(1-p_{N-1}^*)(1-p_{N-2}^*) > S$$

Let us continue and reduce p_{N-1}^* to get a p_{N-1} which satisfies:

$$p_1^* = \ldots = p_{N-2}^*(1-p_{N-3}^*)(1-p_{N-4}^*) > p_{N-1}(1-p_{N-2}^*)(1-p_{N-3}^*) > p_N(1-p_{N-1})(1-p_{N-2}^*) > S$$

We proceed inductively to reduce p_i^*, at each step generating a p_i and preserving two inequalities. After the N-1 step a polite policy is obtained which attains S, thus $C_N \geq S = C_N^* - \epsilon$. We conclude, $C_N \geq C_N^*$.

The second step is to prove the reverse inequality: $C_N^* \geq C_N$.

To derive $C_N^* \geq C_N$ we employ the following idea: we show that any S which is attained by a polite policy P, is attained by a fair policy P; that is, we show that $S < C_N$ implies $S < C_N^*$, from which $C_N \leq C_N^*$ follows.

Let S be attained by a polite policy P, we construct a fair policy P which attains S too, using an iterative "improvement" of P. The limit of the iterations is P.

Let us define an "improvement" operator T over the set of policies $P' = T(P)$:

$$p'_N \triangleq \frac{S}{(1-p_{N-1})(1-p_{N-2})}$$

$$p'_{N-1} \triangleq \text{The solution of} \quad \frac{p'_{N-1}}{1-p'_{N-1}} = \frac{p'_N}{1-p_{N-3}}$$

$$p'_{N-2} \triangleq \text{The solution of} \quad \frac{p'_{N-2}}{1-p'_{N-2}} = \frac{p'_{N-1}}{1-p_{N-4}}$$

and so on.

Claim:

1. *If* (P,S) *is a stable polite system then so is* (P',S).

2. $P' \ll P$, *i.e.*, $p'_i < p_i$ *for all* $1 \leq i \leq n$.

Proof:

To demonstrate (2) we start with the inequality:

$$p_N(1-p_{N-1})(1-p_{N-2}) > S$$

so that

$$p_N > \frac{S}{(1-p_{N-1})(1-p_{N-2})} = p'_N$$

now

$$\frac{p'_{N-1}}{1-p'_{N-1}} \triangleq \frac{p'_N}{1-p_{N-3}} < \frac{p_{N-1}}{1-p_{N-1}}$$

where the rightmost inequality follows from the politeness of P. The last inequality further implies:

$$p'_{N-1} \leq p_{N-1}.$$

Proceeding along the same lines one may derive:

$$\forall\, i,\ 1 \leq i \leq N,\ p'_i < p_i$$

so that the second part of the claim is proved.

Let us turn now to the first part of the claim; to show that it is true we use

$$p'_N(1-p'_{N-1})(1-p_{N-3}) > p'_N(1-p_{N-1})(1-p_{N-2}) = S$$

as our point of departure. The inequality follows from part (2) and the equality from the definition of P'. We may proceed inductively as follows:

$$p'_{N-1}(1-p'_{N-3}) > p'_{N-1}(1-p_{N-3}) = p_N(1-p'_{N-1}) > p'_N(1-p'_{N-1})$$

yields:

$$p'_{N-1}(1-p'_{N-2})(1-p'_{N-3}) > p'_N(1-p'_{N-1})(1-p'_{N-2}) = S$$

and so on. Once we carry the process to the end we have proved that (S,P') is stable.

This completes the proof of the claim; to complete the proof of the lemma let us start with a stable polite tandem (S,P). Consider the iterates of T operating on P: $P^k \triangleq T^k(P)$ then

(i) (S,P^k) is stable and polite.

(ii) $\forall i,\ 1 \le i \le N\ \{p_i^k\}_{k=1}^{\infty}$ is a decreasing sequence bounded from below.

Therefore the limit $p_i \triangleq lim_{(k \to \infty)}\ p_i^k$ exists and $0 \le p_i \le 1$.

It is easy to check that (S,P) is a fixed point of T. Thus, by definition of T, we have:

$$p_1 = p_2(1-p_1) = \ldots = p_{N-1}(1-p_{N-2})(1-p_{N-3}) = p_N(1-p_{N-1})(1-p_{N-2}) = S$$

so that P is a fair policy which attains S. This concludes the proof of the second part, i.e., $C_N \le C_N^*$.

Q.E.D.

5.2.2 HOW TO TUNE UP YOUR TANDEM ?

The lemma of the previous section enables us to compute the capacity of polite (or fair) tandems. All we have to do is maximize the value of S which may be sustained by a fair policy. That is, find the maximal S satisfying:

$$S = p_1(S) = p_2(S)(1-p_1(S)) = \ldots\ldots = p_N(S)(1-p_{N-1}(S))(1-p_{N-2}(S)) \qquad (5.2\text{-}4)$$

A pair (S,P(S)) for which 5.2-4 is satisfied, we call a *tuned-up* tandem. Rewriting the recurrence relations of 5.2-4 as:

$$
\begin{aligned}
& p_1 = S \\
& p_2 = [p_1/(1-p_1)] \\
& p_3 = [p_2/(1-p_2)] \\
& p_4 = (1-p_1) \times [p_3/(1-p_3)] \\
& \quad . \\
& \quad . \\
(5.2\text{-}5) \qquad & \quad . \\
& \quad . \\
& p_i = (1-p_{i-3}) \times [p_{i-1}/(1-p_{i-1}) \\
& \quad . \\
& \quad . \\
& p_N = (1-p_{N-3}) \times [p_{N-1}/(1-p_{N-1})
\end{aligned}
$$

We get a nonlinear system of recurrence relations.

To solve the system 5.2-5 we use a linearization trick. Consider the following system of linear recurrence equations:

$$f_1 = f_2 = f_3 = 1$$

(5.2-6)

$$f_{i+3} = f_{i+2} - Sf_i$$

LEMMA:

For $0 \le S \le 1$ *if* $\{f_j(S)\}_{j=1}^{\infty}$ *is a solution of the linear system* 5.2-6, *then* $p_j(S) \triangleq S[f_j(S)/f_{j+2}(S)]$ *solve the nonlinear system* 5.2-5 *of the tuned-up tandem.*

PROOF:

Let us use induction:

$$p_1(S) \triangleq S, \qquad p_2(S) \triangleq Sf_2/f_4 = S/(1-S) = p_1/(1-p_1).$$

Assume that we have proved already that

$p_1(S), p_2(S), \ldots\ldots\ldots, p_{i-1}(S)$

solve the first $i-1$ equations of 5.2-5; then

(5.2-7) $\quad p_i(S) \triangleq Sf_i/f_{i+2} =$

so that the i-th equation is satisfied too and the proof is concluded.

Q.E.D.

Now that the tuning-up problem had been linearized the path to follow is the routine z-transform solution . We define

(5.2-8) $\quad F(z,S) \triangleq \sum_{j=1}^{\infty} f_j(S)\, z^j$

We use the notation F(Z) when there is no danger of confusion. The *transformation of equation 5.2-6 yields:*

$$(5.2\text{-}9) \qquad F(z) \triangleq \frac{f_1 z + z^2(f_2 - f_1) + z^3(f_3 - f_2)}{Sz^3 - z + 1}$$

Finally:

$$(5.2\text{-}10) \qquad F(z) = \frac{z}{Sz^3 - z + 1}$$

Which may be inverted to give:

$$(5.2\text{-}11) \qquad f_j(S) = \sum_{k=0}^{\lfloor (j-1)/3 \rfloor} (-S)^k \binom{j-2k-1}{k} \qquad j \geq 1$$

Therefore the tuning-up problem is solved. In table 5.2-1 below we list few of the polynomials $f_j(S)$. (Note the surprisingly nice structure of the polynomials $f_j(S)$. The coefficients of $(-S)^k$ are the partial sums of the coefficients of $(-S)^{k-1}$.)

Figure 5.2-2 depicts the tuned-up policies $p_j(S)$ versus S.

$f_1(S) = f_2(S) = f_3(S) = 1$

$f_4(S) = 1 - S$

$f_5(S) = 1 - 2S$

$f_6(S) = 1 - 3S$

$f_7(S) = 1 - 4S + S^2$

$f_8(S) = 1 - 5S + 3S^2$

$f_9(S) = 1 - 6S + 6S^2$

$f_{10}(S) = 1 - 7S + 10S^2 - S^3$

$f_{10}(S) = 1 - 8S + 15S^2 - 4S^3$

$f_{10}(S) = 1 - 9S + 21S^2 - 10S^3$

$f_{15}(S) = 1 - 10S + 28S^2 - 20S^3 + S^4$

... ...

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\ \dots \; \dots \; \dots \; \dots \; \dots \; \dots \end{array}$$

Table 5.2-1: The polynomials $f_j(S)$

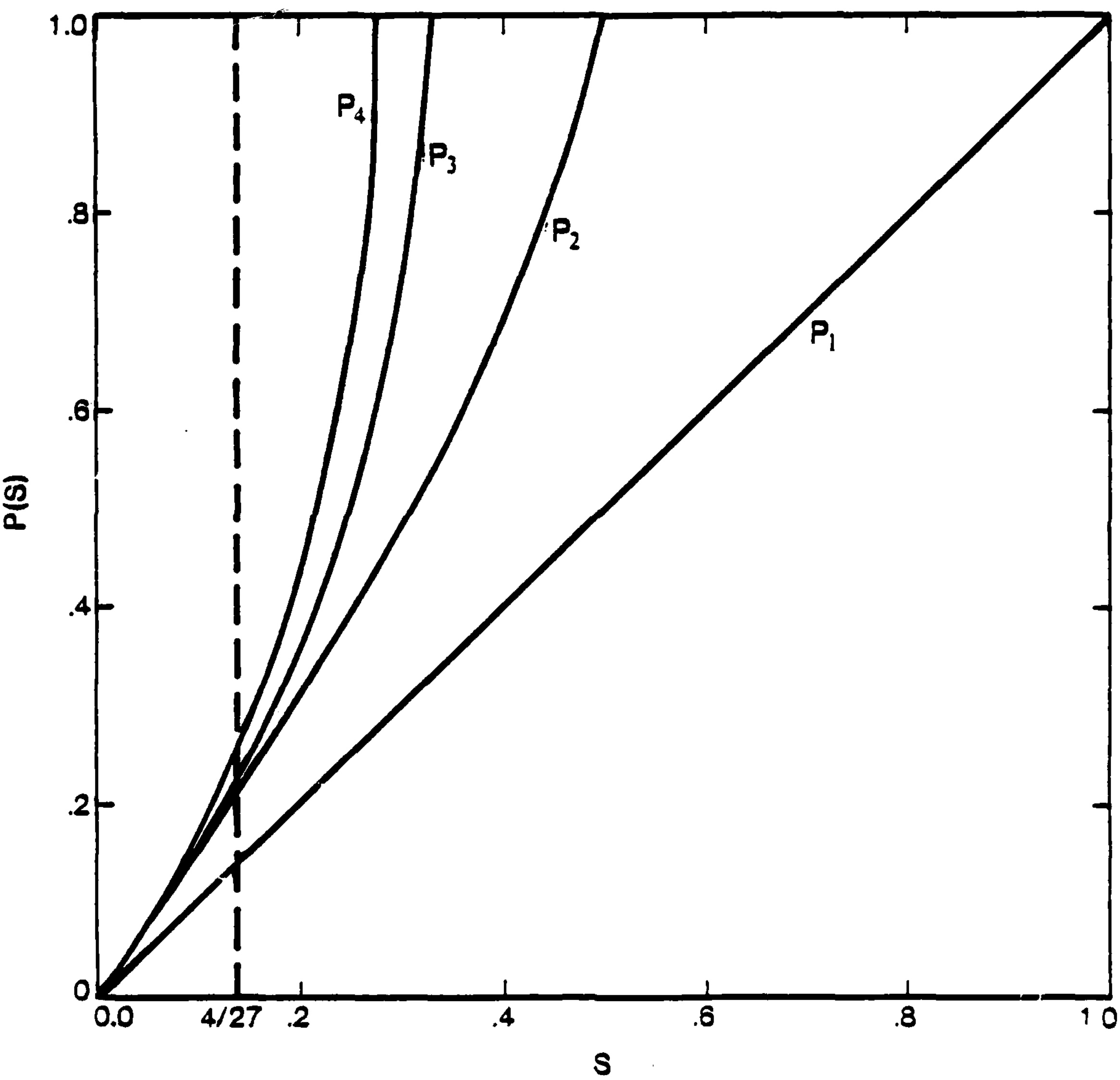

Figure 5.2-2: Tuned-up transmission policies

THE CAPACITY OF POLITE TANDEMS (HEAVY-TRAFFIC CAPACITY)

In this section we derive a solution to the capacity problem for polite tandems.

LEMMA:

Let (S,P(S)) be a tuned-up tandem, then:

(A) $p_1(S) < p_2(S) < \ldots\ldots < p_N(S)$

(B) *For* $1 \le i \le N$, $p_i(S)$ *is an increasing function of S.*

PROOF:

(A)

$$p_2 = p_1(1 - p_1) > p_1$$

$$p_3 = p_2(1 - p_2) > p_2$$

so

$$p_1 < p_2 < p_3$$

Proceed by induction. Let us assume that:

$$p_1 < p_2 < p_3 < \ldots\ldots\ldots < p_i$$

then

$$p_{i+1} = p_i \times [(1-p_{i-2})/(1-p_i)] > p_i$$

Thus $p_{i+1} \ge p_i$ and (A) follows.

(B)

The claim of (B) clearly holds for $p_1(S) = S$ and $p_2(S) = S/(1-S)$ (i.e., both are increasing functions of S in [0,1]). Let us use induction again. Assume: $p_1(S)$, $p_2(S)$,.....$p_i(S)$ are all increasing functions of S. now

$$p_{i+1}(1-p_i)(1-p_{i-1}) = S \text{ implies } p_{i+1}(S) = S/[(1-p_i(S))(1-p_{i-1}(S))]$$

When S increases the induction assumption guarantees that the right hand side increases too. This concludes part (B).

Q.E.D

THEOREM:

The capacity of the tandem of length N C_N *is the minimal zero of the polynomial* $f_j(S)$ *in* [0,1].

PROOF:

If (S,P(S)) is a tuned-up tandem and we let S grow from 0 to C_N, then P(S) will grow coordinatewise and

$$0 < p_1(S) < p_2(S) < \dots\dots\dots < p_N(S) < 1$$

The polite capacity C_N is the maximum value of S which is attainable in a tuned-up system. This is the value of S for which $p_N(C_N) = 1$ This equality further implies

$$1 = C_N f_N(C_N) / f_{N+2}(C_N)$$

namely

$$0 = f_{N+2}(C_N) - C_N f_N(C_N) = f_{N+3}(C_N)$$

Q.E.D

Using the above theorem, it is easy to compute C_N. In Figure 5.2-3 We plot the capacity C_N of polite tandems versus their length N.

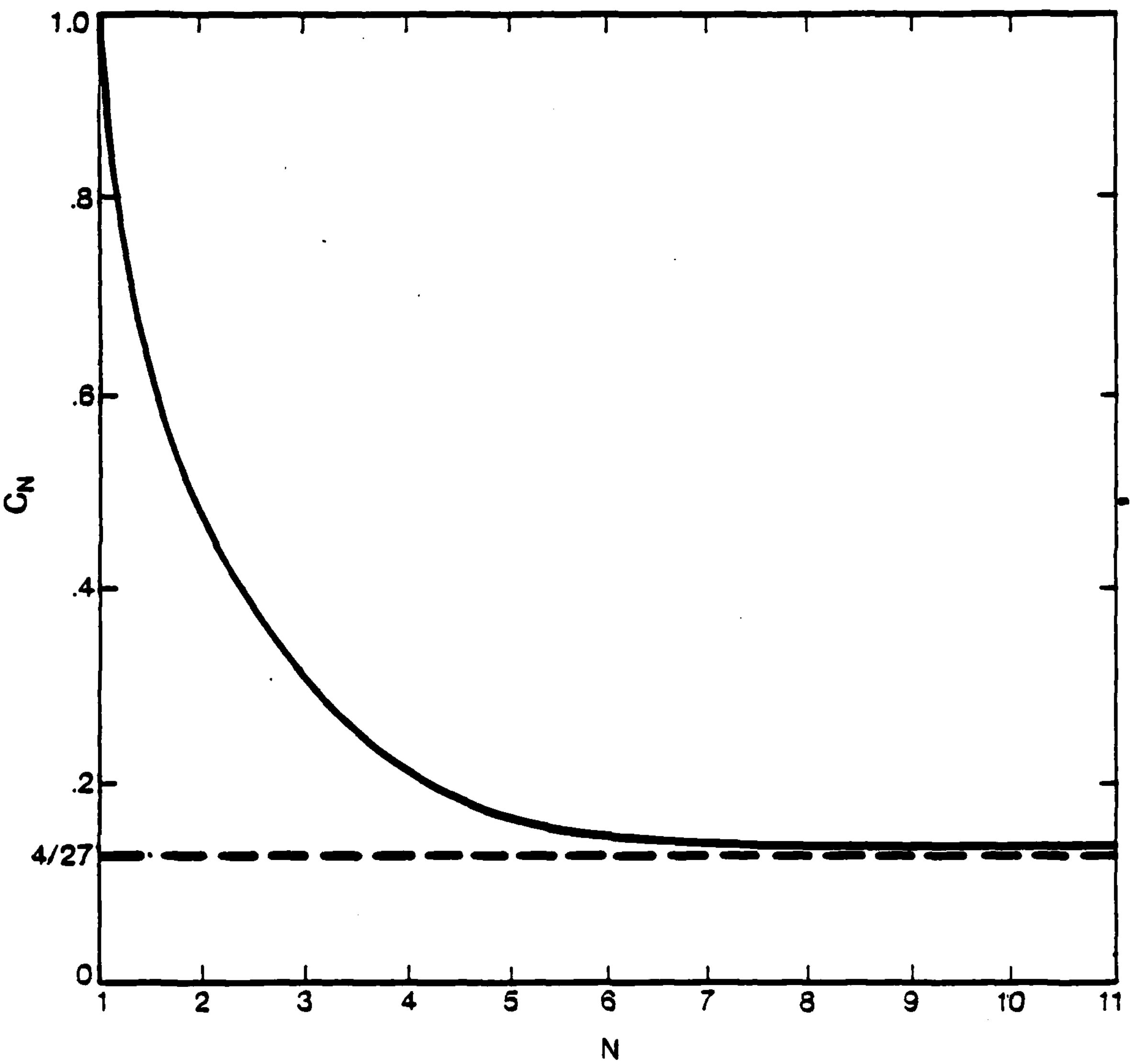

Figure 5.2-3: Heavy-traffic (polite) capacity of tandems

5.2.3 ONE, TWO, THREE, FOUR, INFINITY !

One hop ALOHA PRNETs tend to become unstable as they grow in size [KLEI76, LAM74, FERG75]. The infinite population system, so to speak, can produce so much cross interference that no open loop transmission policy will stabilize it.

Tandems are "nicer" PRNETs in this respect. The amount of interference does not depend upon the system size. One expects some better limiting behavior. Can an infinite tandem be stable ? What is the polite capacity of an infinite tandem ?

CLAIM:

(A) $C_{\infty} \triangleq 4/27$, *is an attainable throughput by a tandem of any length.*

(B) C_{∞} *is the largest such throughput.*

(C) *For all* $i \geq 0$, $p_i(C_{\infty}) < 1/3$.

PROOF:

(A)

Let us return to the expression 5.2-10 for the transform $F(z,S)$ of the sequence $\{f_j(S)\}_{j=1}^{\infty}$.

$$F(z,4/27) = z/[(1+z/3)(1-2z/3)^2]$$

Inverting the transform we get:

$$f_j(4/27) = (1/9)[\,(-1/3)^{j-1} + 9j(2/3)^j + 2(2/3)^{j-1}\,]$$

Clearly $\forall\, j \geq 1$ $f_j(4/27) > 0$. Thus by the theorem of section 5.2.2 on page 239, $C_j > 4/27$ for all $1 \leq j$, which concludes the proof of part (A).

(C)

To derive the inequality it is enough to show:

$$\forall\, j \geq 1,\ (1/3) > (4/27)[\,f_j(4/27)\,/\,f_{j+2}(4/27)\,]$$

This follows through some simple uninteresting tedious manipulations which we omit.

(B)

Finally, let S be any throughput attainable by an infinite tandem. $\{p_j(S)\}_{j=1}^{\infty}$ is an increasing sequence of numbers bounded from above. Let $p_j^*(S) \triangleq lim_{(j \to \infty)}\, p_j(S)$; $S = p_j(1-p_{j-1})(1-p_{j-2})$ for $j \geq 4$ implies the limiting relation: $S(p^*) = p^*(1-p^*)^2$. The maximal value of S is: $\max\{S(p^*)\ |\ 0 \leq p^* \leq 1\}$, which is 4/27.

Q.E.D

The polite capacity of an infinite tandem is 4/27. The tuned-up policy that attains this capacity satisfies $p_j(4/27) < 1/3$ for $j \geq 1$. The convergence to the limiting behavior is fast. Indeed, the polite capacity C_N drops as N increases according to the rule: one, two, three, four, infinity (i.e., for N>4 C_N is already almost 4/27).

5.3 RUDE TANDEMS ARE BETTER

Consider the tandem PRNET of the previous section. Let us assume that the tandem chooses the policy $P \triangleq (1,1,\ldots\ldots,1)$; we call this policy a "*rude*" policy. Indeed each PRU is as inconsiderate towards the needs of his fellow PRUs as possible.

Fortunately, being maximally inconsiderate improves the performance of the tandem by far. An arriving packet is queued at the upper end of the tandem waiting for its turn to be relayed forward. Once it enters the tandem it is guaranteed a free channel at each hop. The phased propagation of packets is depicted in Figure 5.3-1. The effects of rudeness are identical to the behavior of the "maximal interference" model for two interfering PRUs (chapter 3.4).

The rude tandem provides the best possible service - a perfect scheduling of the channel resource. The capacity it obtains is 1/3. This is the best capacity that a multi-hop PRNET may obtain. Indeed, in a multi-hop network the service of a packet at any single node requires at least 3 slots. Thus, at most one third of the channel may be utilized for actual work. The rude tandem obtains this upper bound.

The rude policy has some additional features of importance. Rudeness introduces a natural flow control mechanism which restricts the number of packets that may exist in the network at any single slot to a minimum. The rude tandem does not require excessive buffering inside the network. It offers the best network behavior which we may hope for.

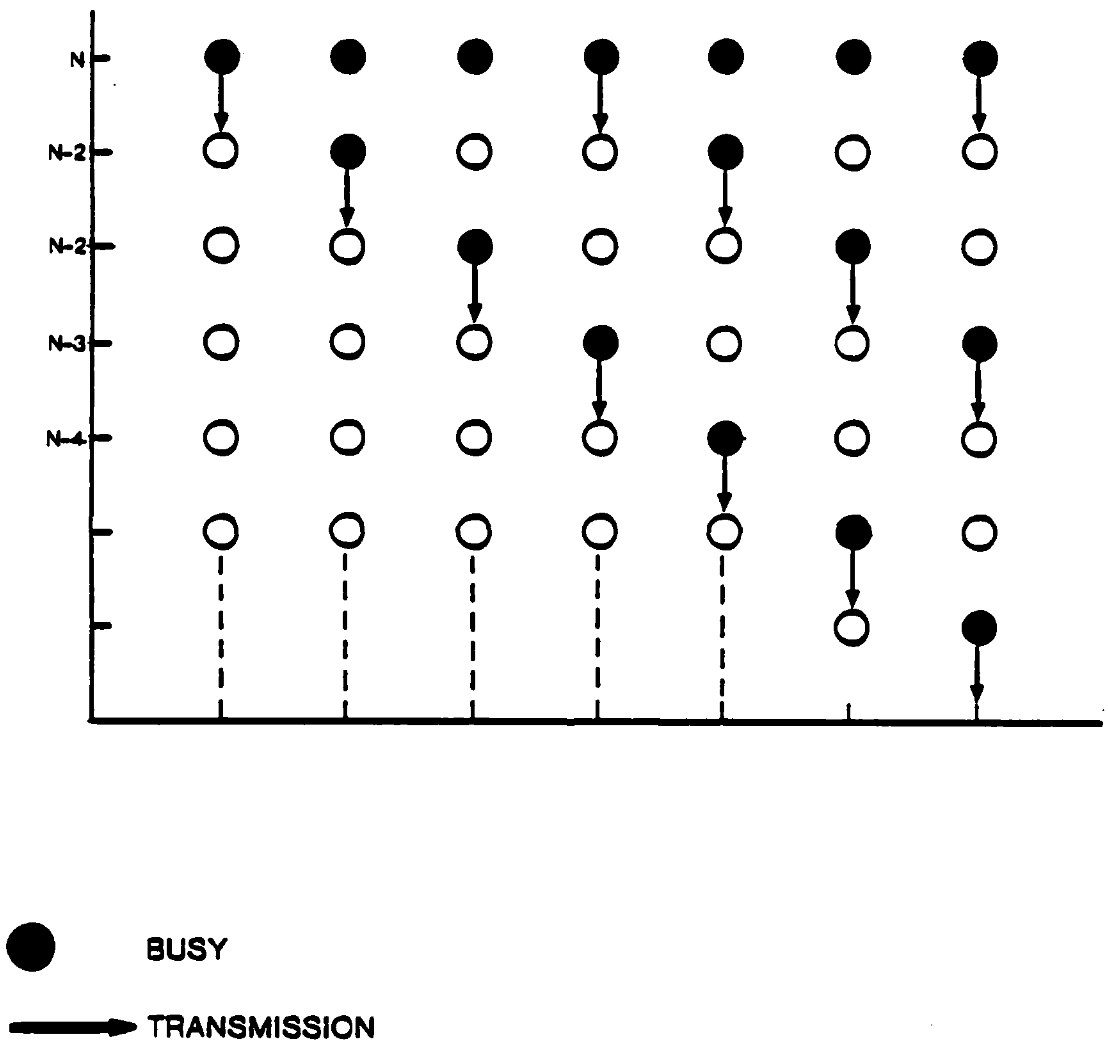

Figure 5.3-1: Phased packet propagation on a "rude" tandem

5.3.1 THE COMBINATORICS OF RUDENESS (PHASING IN NETWORKS)

In this section we would like to develop some better insight into the problem of singularity. What makes rudeness possible? How well can a singular network perform? Are there other networks which are singular?

Let us reconsider the tandem. The ability of a rude policy to perform perfectly results from two singular properties:

1. A collision on a tandem does not lead to a deadlock.

2. It is possible to generate configurations of non interacting packets whose movement is synchronized.

To generate singular networks we should design the hearing topology to obtain the above features. NOTE, our point of departure is in no way prescriptive. That is, from a practical point of view, the hearing topology is hardly a design parameter that may be chosen. All we want is to understand singularity.

Let us start with a tandem and increase the transmitting power twice, thrice, etc... Figure 5.3-2 depicts the hearing topology of the resulting hearing graphs; we are getting overlapping tandems. It is easy to check that when the arrivals are restricted to the upper ends of the different subtandems, rudeness leads to the generation of perfectly synchronized trains of moving packets. Figure TRAINS describes some typical trains. The network becomes a natural scheduling mechanism which eliminates collisions even at the last hop. The throughput that such networks obtain is $n/2n+1$, where n is the increase in power. The limiting throughput is $1/2$. This indeed can be easily shown to be

the best possible throughput for any multihop network with a fully connected hearing graph near the station.

Therefore, singular network structures, other than the tandem, may exist. Moreover, such network structures may be used to synchronize the random demands of communication, perfectly. In the context of a PRNET perfect singularity may not exist, yet the discussion above shows that singular hearing topology is a desirable feature to strive at. To a large extent this may be acheived by using, for instance, directional antennas, or taking advantage of the geography.

The optimality of rude transmission policies results from the singularity of both the topology and the input structure. Consider the tandem with arrivals distributed all along. The rude behavior can no longer create those nice trains of non- interferring packets as in Figure 5.3-1. If we consider the busy status of the queues in the tandem, it is clear that rudeness is going to be a very bad policy during many slots. For instance, if the tandem is kept fully busy most of the time then politeness is superior to rudeness.

Between rudeness and politeness there is a spectrum of admissible transmission policies. Which policy should the network choose ?

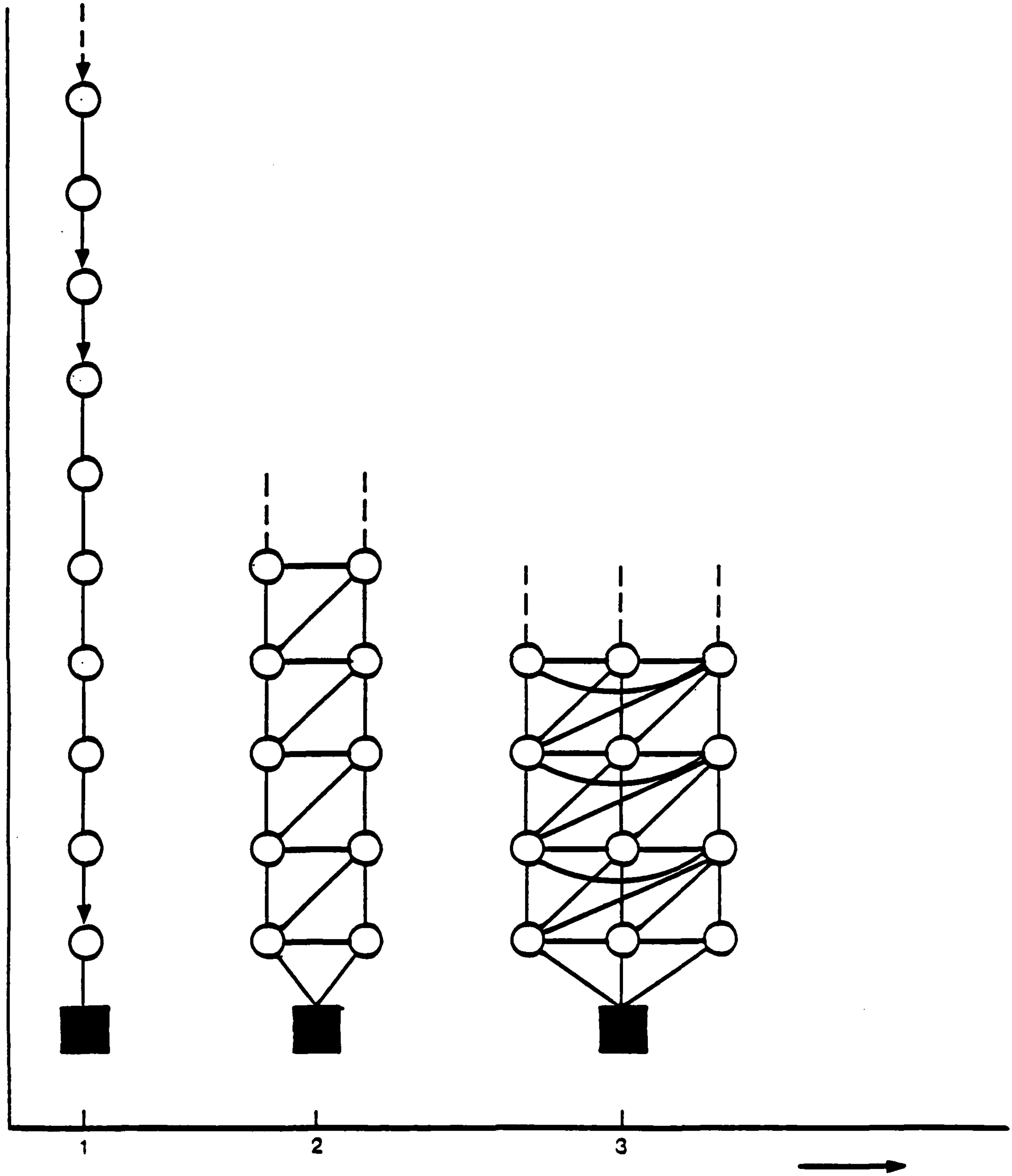

Figure 5.3-2: Singular PRNETs generated from a tandem by power increase

5.4 FROM RUDENESS TO PRUDENCE

(THE ECOLOGY OF COMMUNICATION CHANNELS)

5.4.1 PARETO POLICIES FOR SHARING THE COMMUNICATION CHANNEL

In what follows we shall develop a novel attack upon the capacity problem. The method that we employ generalizes that of section 5.1.2. We shall reconsider the problem of decentralized decision from a fresh point of view, develop a suitable notion of decentralized optimality criteria and characterize the optimal policies w.r.t. this criteria. The rules of behavior which are so developed can be easily implemented as an effective decentralized transmission control algorithm which requires only acknowledgement traffic to reach decisions. Both optimal transmission control policies, optimaly controlled Slotted-ALOHA and the optimal random Urn scheme turn out to be particular realizations of the optimality criteria which are developed.

Let us consider the problem of choosing a transmission policy from the point of view of an individual PRU. If he chooses to transmit with a high probability he may be wasting the shared channel, polluting it with collisions. If his transmission probability is too small, he may be wasting useful slots in silence. The PRU faces a typical problem of ecology, i.e., how to best utilize the shared natural resource. What is an optimal choice of transmission probabilities?

The question may be answered only if we impose some global objective function to be optimized by the network. The decentralization of the decision process and the requirement of a real-time decision mechanism may rule out the possibility of implementing a global optimization mechanism. The classical approach to the problem is to model the constraints quantitavely and solve the

constrained optimization problem. This approach seems to lead to insurmountable difficulties when it comes to a characterization of the constraints imposed by decentralization, computational and communication complexity and the real-time constraint. Even a simple description of the above constraints leads to intractable optimization problems. Alternative approaches should be sought.

One possible alternative is to develop a (sort of) dual approach. That is, rather than keeping a hard-to-reach objective fixed and dealing with an extremely restricted set of available policies, we may keep the set of policies fixed and choose a restricted objective function. The idea is simple: if the objective is too hard to reach let us replace it with an easier objective, i.e., weaken it. The weakening process must be consistent, i.e., the set of solutions to the weaker optimization problem should include the solutions to the original problem. An additional merit to this weakening process is that the class of weaker solutions may include policies which are suitable for different objectives (for instance minimizing delay, maximizing throughput).

Let us consider the problem of finding a suitable weaker objective function. A trivial solution is to consider as a weak notion of optimality no notion at all. The set of weakly optimal solutions is the set of all policies. In particular any solution to a global optimization problem belongs to the set of weak solutions. Of course this is an uninteresting solution.

To generate an interesting notion of optimality we should put ourselves again in the *shoes* of an individual PRU. As far as he is concerned, the quality of the shared resource can be measured in terms of the expected number of successful

packets that he may deliver, i.e., the individual throughput. A reasonable choice of policy must be such that it is impossible for all PRUs to improve their individual utility (throughput) simultaneously. Recall that policies satisfying this condition are called *Pareto optimal*. This is a reasonable choice of a weak sense of optimality. Indeed, it is a suitable notion for any global objective compatible with the individual utility function of a PRU. However, the usefulness of this notion can only be appreciated once we characterize optimal policies.

The choice of a weaker notion of optimality presented above is quite arbitrary and could only be justified after the fact. An important question to consider is whether it is possible to produce a parametrized system of objective functions which is monotonic in the parameter (in the sense of set inclusion of optimal policies). Is it possible to translate the constraints of distributivity into constraints upon the parameter of the objective functions?

We leave the above question for a future research, proceeding with a formalization of the heuristic utility function which we described above. Let $\underline{b}^t \triangleq (b_1^t, b_2^t, \ldots, b_N^t)$ designate the occupancy process, that is:

$$(5.4\text{-}1) \qquad b_i^t \triangleq \begin{cases} 1 & \text{if } PR_i \text{ has a ready packet at slot } t. \\ 0 & \text{otherwise.} \end{cases}$$

Let $\pi^t(\underline{b})$ designate the distribution of $\underline{b}^t$.

We define the throughput process:

$$(5.4\text{-}2) \qquad s_i^t \triangleq \begin{cases} 1 & \text{if } PR_i \text{ successfully delivers a packet to his destination, at slot } t. \\ 0 & \text{otherwise.} \end{cases}$$

Let us assume that each node PR_i routes its packets to a single destination $PR_{d(i)}$. The routing matrix R describes a spanning tree of the hearing graph. We can express the mean throughput of PR_i, conditioned upon $\underline{b}^t = \underline{b}$, as follows:

$$(5.4\text{-}3) \qquad s_i^t(\underline{b},P) \triangleq E[s_i^t \mid \underline{b}^t = \underline{b}] = \begin{cases} p_i \prod_{j \in I_i(\underline{b})} (1-p_j) & \text{if } b_i = 1 \\ 0 & \text{if } b_i = 0 \end{cases}$$

where

$$I_i(\underline{b}) \triangleq \{j \mid j \neq i,\ b_j = 1 \text{ and } PR_j \text{ interferes with } PR_j\}$$

The expected throughput of PR_i at slot t, is given by:

$$(5.4\text{-}4) \qquad \bar{s}_i^t(\underline{p}) = \sum_{\underline{b} \in \{0,1\}^N} \pi(\underline{b})\ s_i^t(\underline{b},\underline{p})$$

where $\pi^t(\underline{b})$ is the distribution of $\underline{b}^t$.

Henceforth we shall eliminate the time indexing t, whenever no danger of confusion arise, in order to simplify the notations. Equation 5.4-4 defines a continuous map $S = \underline{S}(P)$ of policies onto attainable throughputs at slot t. The hypercube of policies $\Lambda \triangleq [0,1]^N$ is mapped onto a compact domain $\underline{S}(\Lambda)$ of all attainable throughputs.

Recall the definitions of section 5.1.2; the map $S = \underline{S}(P)$ is called the *Abramson throughput operator*. The throughput $S_i(P)$ is considered as the *utility* which PR_i attaches to the policy P. A throughput vector S is called *Pareto-optimal* if it is not dominated by any other attainable throughput. A policy P, which obtains a Pareto-optimal throughput, is called *Pareto-optimal policy*. Pareto-optimal policies are precisely those policies for which we cannot improve the utility of one PRU without decreasing the utility of some fellow PRUs. Therefore by choosing this notion of optimality we replace a global objective function with a set of local objective functions, easier to handle by a decentralized decision mechanism.

5.5 SILENCE IS GOLDEN.....SO ALSO IS THROUGHPUT

In this section we derive a formal characterization of Pareto optimal policies. The results have a surprising intuitive interpretation.

Let us reconsider the Abramson map defined by equation 5.4-4 of the previous section:

(5.5-13) $S = \sum_{\underline{b} \in \{0,1\}} \pi(\underline{b})\, s(\underline{b},P)$

let P^0 be a Pareto optimal policy obtaining a throughput $S^0 \triangleq \underline{S}(P^0)$. Let P be a policy differing from P^0 by a small perturbation $\Delta P \triangleq P - P^0$. If S is the throughput obtained by P then S constitutes a small perturbation of S^0; let $\Delta S \triangleq S - S^0$ be this perturbation.. The conditional throughputs $s_i(\underline{b},P)$ are smooth functions of P. Therefore ΔS is related to ΔP through a linear transformation:

(5.5-14) $\Delta\underline{S} = \partial\underline{S}\, \Delta\underline{p}$

described by the Jacobian matrix:

(5.5-15) $\partial\underline{S} \triangleq \sum_{\underline{b}} \pi(\underline{b}) \dfrac{\partial\underline{S}(\underline{b},\underline{p}^0)}{\partial\underline{p}} = [\sum_{\underline{b}} \pi(\underline{b}) \dfrac{\partial s_i(\underline{b},\underline{p}^0)}{\partial p_j}]_{ij}$

which we call the Jacobian matrix of the network. The transformation described by 5.5-2 is a linear approximation of the nonlinear Abramson map near the Pareto optimal policy P^0.

Let $D(P^0)$ denote the set of admissible perturbations of P^0. Clearly, $0 \in D(P^0)$. Moreover, if P^0 is an internal point of the set of admissible policies then $D(P^0)$

then the image of $D(P^0)$, i.e., the set of admissible perturbations of S^0 contains a neighborhood of zero. This contradicts the extremality of S^0. Therefore, the Jacobian matrix of the network must be singular at P^0.

What if P^0 is not an internal point of Λ? If P^0 is internal to any face of Λ then the argument above may be repeated by properly restricting the Abramson operator to a subnetwork of the original network. The demonstration involves some lenghty combinatorial arguments that are of no interest for us. Therefore we shall proceed to derive necessary conditions for Pareto optimality, assuming that P^0 is internal to Λ. The conditions that we derive may be verified to hold when P^0 is not an internal point of Λ and even when P^0 is an extreme point of Λ.

We conclude: *if* P^0 *is a Pareto-optimal policy obtaining a throughput* S^0 *then the Jacobian determinant of the network must be zero. That is*

$$(5.5\text{-}4) \qquad 0 = \left| \sum_{\underline{b}} \pi(\underline{b}) \frac{\partial s_i(\underline{b}, \underline{p}^0)}{\partial p_j} \right|$$

The generic elements of the Jacobian are easily computed to be:

$$(5.5\text{-}5) \qquad \partial \underline{S}_{ij} \triangleq \begin{cases} \dfrac{E_i}{1-p_i} & i=j \\[2em] \dfrac{S_{i/j}}{1-p_j} & i \neq j \end{cases}$$

where

$$E_i \triangleq \sum_{\{\underline{b} \mid b_i = 1\}} \pi(\underline{b})(1-p_i) \prod_{j \in I_i(\underline{b})} (1-p_j)$$

is the *expected number of slots which are empty* at the destination of PR_i given that it is busy, and

$$S_{i/j} = \begin{cases} \sum\limits_{\{\underline{b} \mid b_i = b_j = 1\}} \pi(\underline{b})\, p_i \prod\limits_{k \in I_i(\underline{b})} (1-p_k) & \text{if } j \text{ interferes with } i \\ 0 & \text{if } j \text{ does not interfere with } i \end{cases}$$

is the *expected number of successful packets* that PR_i delivers, given that PR_j is busy and that PR_j interferes with PR_i; it is zero otherwise.

Let us consider a typical one-hop network. In this case the optimality condition 5.5-4 may be expressed as:

$$(5.5\text{-}6) \qquad 0 = |\partial \underline{S}| = \begin{vmatrix} E_1 & -S_{1/2} & -S_{1/3} \cdots\cdots & -S_{1/N} \\ -S_{2/1} & E_2 & -S_{2/3} \cdots\cdots & -S_{2/N} \\ -S_{3/1} & -S_{3/2} & E_3 \cdots\cdots & -S_{3/N} \\ \cdots & \cdots & \cdots & \cdots \\ -S_{i/1} & -S_{i/2} \cdots\cdots & E_i \cdots\cdots & -S_{i/N} \\ \cdots & \cdots & \cdots & \cdots \\ -S_{N/1} & -S_{N/2} \cdots\cdots & \cdots\cdots & E_N \end{vmatrix}$$

In particular consider the symmetric case when all PRUs are alike, the determinant becomes (after some easy algebra):

$$(5.5\text{-}7) \qquad |\partial S| = [\hat{E}-(N-1)\hat{S}]\,[\hat{E}+\hat{S}]^{N-1}$$

where $\hat{E} \triangleq E_1 = E_2 = \ldots = E_N$ and $\hat{S} \triangleq S_{i/j}$ for all i and j $(i \neq j)$.

This expression is zero iff:

$$(5.5\text{-}8) \qquad \hat{E} = (N-1)\hat{S}$$

The left hand side represents the expected number of slots that a busy PRU leaves empty at the destination: we call this: "*silence*". The right hand side represents the amount of throughput, produced by the rest of the network, that a busy PRU sees; we call this: "*throughput*". Thus, a *necessary condition for an optimal selection of transmission policies is that each PRU equate "silence" with "throughput"*.

We conclude for the symmetric one-hop PRNET that if a policy is Pareto-optimal, then each busy PRU trades the slots which he wastes in silence for an equal number of slots successfully used by all those PRUs which may be harmed by his transmission.

Let us return to the determinant of the general network 5.5-4. If it is to be zero, there should be a linear combination of its rows which yields zero. Let us denote the coefficients of such a linear combination $\underline{c} = (c_1, c_2, \ldots\ldots\ c_N)$. The optimality condition:

$$c_1E_1 = \sum_{\{j \mid 1 \in I(j)\}} c_jS_{j/1}$$

$$c_2E_2 = \sum_{\{j \mid 2 \in I(j)\}} c_jS_{j/2}$$

$$\vdots$$

$$c_NE_N = \sum_{\{j \mid N \in I(j)\}} c_jS_{j/N}$$

may be interpreted as follows. Each PRU PR_i is endowed with a slot dollar cost c_i reflecting his relative significance in the network. The left hand side of each of the above expressions represents the value of silence of the respective PR_i. The right hand side represents the value of success for the busy PRUs whose transmission PR_i may harm, given that PR_i is busy. *The optimality condition requires that each PRU, once busy, should consider the needs of his fellow PRUs. He should be ready to trade his silence for an equivalent dollar-worth amount of their throughput, by selecting his transmission policy to balance the two quantities.*

Therefore by properly choosing a pricing policy for the network we may approximate global objectives through the decentralized optimization mechanism. For instance, if we permit dynamic pricing than it is possible, if we have centralized perfect scheduling mechanism, to give some PRUs an infinite cost and other PRUs a zero cost; by properly choosing the allocation of prices, perfect scheduling is obtained. If we do not have a perfect information pricing mechanism we may still adjust prices every once in a while or make the prices time dependent. Various interesting decision mechanisms may be obtained.

It seems that by properly combining the decentralized optimality criteria with some hierarchical adaptive pricing mechanism, we could develop an

answer to the problem of time-decision hierarchy which was raised in the second chapter. The precise formulation of the answer, as well as the problem of establishing the relation between global objectives and different pricing mechanisms, are left for future research.

Let us consider the ecology of the communication channel. This shared resource may be wasted by collisions if the users are too rude. If the users are too polite they may leave much of the resource under-utilized. The Pareto-optimal policies make the best expected use of the channel in the sense that every slot wasted in silence by one PRU is utilized by another. The pollution of the channel by collisions or under-utilization, during some slots, is not the result of imprudent network behavior, but the consequence of the statistical fluctuations of the service demands. *Pareto-optimality, thus, represents maximal prudence in the face of lady luck.*

Now we explore the relation of the characterization which we obtained above to the heavy-traffic results of Abramson, presented in section 5.1.2. We examine the asymptotic behavior of a Pareto-optimal one-hop network as the traffic becomes heavy. The heavy-traffic condition may be expressed in terms of the distribution of occupancy. Namely, $\pi(\underline{b}) = \delta(x,\underline{1})$, where $\underline{1}$ is the vector all of whose coordinates are 1 and δ is Dirac's delta distribution. The Jacobian matrix of the network degenerates into the form which had been considered in section 5.1.2 . The optimality conditions reduce to the conditions of equation 5.1-5 in section 5.1.2. We see that Abramson's results are a particularization of our characterization when the traffic is very heavy.

Finally we wish to show that the optimal Urn policy derived in chapter 2,

satisfies the optimality condition 5.5-6. Let us assume that n PRUs out of N network members are busy. Let k be the number of PRUs selected for transmission. We consider the decision making from the point of view of a given busy PRU. "silence" occurs if all k owners of transmission right happen to be other non busy PRUs. The probability that this occurs, given that our PRU is busy, is:

$$E_1 = \frac{\binom{N-k-1}{n-1}}{\binom{N-1}{n-1}} \tag{5.5-9}$$

The probability of a successful use of a slot by another PRU, given that our designated PRU is busy, is given by:

$$S_1 = \frac{\binom{k}{1}\binom{N-k-1}{n-2}}{\binom{N-1}{n-1}} \tag{5.5-10}$$

Equating the two quantities we find that k should satisfy

$$\frac{1}{n-1} = \frac{k}{N-k-n+1}$$

from which follows

(5.5-11) $k = (N-n+1)/n$

This is precisely the optimal choice of k as given by equation 2.2-19 of section 2.2.2. Therefore, a busy PRU should select k in such a way so as to equate his expected silence with the throughput of others.

5.5.1 DECENTRALIZED ALGORITHMS TO OPTIMIZE TRANSMISSION POLICIES

The characterization of optimal transmission policies, obtained in the previous section, may serve as a basis for a set of distributed access control algorithms. The algorithms are quasi-static in the sense of [GALL77]. The decision making is completely decentralized. The information required for a decision is available to each PRU, at no extra cost, through the acknowledgment mechanism.

The algorithms consist of a decentralized iterative process that tries to balance "throughput" and "silence". The problem is essentially that of solving a large stochastic system of balance equations through gradient iterations. The details of the algorithms, as well as the problems of convergence, are beyond the scope of this dissertation and are left for future research.

Let us recall the optimality rule: For each PR_i

(5.5-12) $\hat{E}_i = \hat{S}_i$

where $\hat{E}_i$ is the expected dollar-value of empty slots at the destination of PR_i, when PR_i is busy. $\hat{S}_i$ is the expected dollar-value of the throughput of those PRUs with which PR_i may interfere when it is busy.

The algorithm to implement the rule 5.5-12 consists of

1. ESTIMATION:

   Each PRU gathers acknowlegments statistics during his busy periods. We assume that both successful packet deliveries and

collisions are acknowledged. PR_i can monitor the acknowledgments sent by all PRUs that he may hear. Therefore PR_i observes s_j^t for all PR_j such that $i \in I(j)$, from which he may compute $S_i^t \triangleq \sum c_j^t s_j^t$, where the summation extends over all PR_j with which PR_i may interfere ($i \in I(j)$). The conditional expectation of S_j^t given that PR_i is busy, is precisely the "throughput" that we wish to estimate. The estimation of the last parameter can follow standard methods for estimating point processes [SEGA76].

Similarly, "silence" can be estimated by monitoring the acknowledgements sent by $PR_{d(i)}$* (in fact, by monitoring slots in which d(i) does not acknowledge anything). Again the problem is that of estimating the conditional expected value of an observed point process.

If only successful packets are to be acknowledged, then E_i may be estimated from the unconditional expected silence at $PR_{d(i)}$ to be monitored by the latter. If the acknowledgement mechanism is not collision free, further sophistication must be introduced into the estimation mechanism.

2. ADAPTATION:

Here the rule is simple: if $E_i > S_i$, then PR_i knows that he wastes too many slots, which nobody else uses anyway, in silence. PR_i

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*Here d(i) denotes the immediate destination of transmissions from PR_i.

will increase p_i. If $E_i < S_i$ then, by the same token, PR_i knows that he is talking too much, preventing fellow PRUs from getting a fair portion of the channel. PR_i should decrease p_i.

There is only one problem: the rule 5.5-12 is necessary but insufficient. For example, the rude policy $P \triangleq (1,1,\ldots 1)$ satisfies the rule independently of the input structure giving $E_i = S_i = 0$. Clearly, our algorithm may lead the network to choose this policy even when the results are disastrous. We have to design some precautionary measures to prevent our algorithm from converging to the rude policy when it should not*.

The required modifications are simple: Each PRU monitors his own throughput. If, as a result of an increase in p_i, PR_i watches his throughput dropping, he knows that he may have increased it beyond the optimal value. The natural response is to decrease the value of p_i.

The above control mechanism may be implemented in a similar fashion to the control mechanisms presented by L. Kleinrock and M. Gerla in [KLEI77, GERL77]. Moreover, when the traffic becomes heavy the algorithm will be an implementation of Abramson's optimality criteria. Thus, it will become identical to those mechanisms.

In a similar fashion we may use the optimality rule to implement a version of the Urn scheme. Indeed, as demonstrated in the previous section, the optimal selection of k (the number of PRUs possessing a transmission right) satisfies the

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*Note, however, that a "rude" policy may sometimes be the best policy, e.g., in the case of a singular network topology.

optimality rule. Therefore, k may be adjusted using the information acquired from the acknowledgment traffic only. The algorithm to implement the scheme consists of two parts similar to those above

1. ESTIMATION: same as before.

2. ADAPTATION:

If $E_i < S_i$, then using the expressions of the previous sectionfor E_i and S_i (5.5-9 and 5.5-10), we find that

$$(N-n+1)/n < k$$

That is, k is too large; PR_i should lower his estimate of k. Similarly, if $E_i > S_i$, PR_i should increase his estimate of k.

By combining the above decentralized decision mechanism with a higher-order pricing mechanism, it is possible to establish priority mechanisms over the network. This possibility of establishing a family of hierarchical resource-sharing mechanisms is an appealing subject for further research and experiments. Further work is also required to develop the details of the estimation and adaptation mechanisms, prove the convergence of the algorithms, compare their performance in the context of one-hop systems with that of known control schemes and test them in a multi-hop environment.

To conclude the discussion, we have seen that Pareto-optimality provides an excellent norm for rational decentralized resource-sharing. Using this criterion, we have derived an intuitively simple, yet powerful, rule-of-thumb for optimal, decentralized, multi-access control. Not only does the optimality

rule encompass previous results (e.g., optimal Slotted-ALOHA and the optimal Urn Scheme), but it also enhances our understanding of proper hierarchical, decentralized, channel-allocation policies. Moreover, it yields a class of simple distributed decision mechanisms to implement the optimal policies. Further research is required to explore the stability of the proposed access-control mechanisms and to apply our methods to solve other problems of decentralized resource-sharing in computer communication networks.

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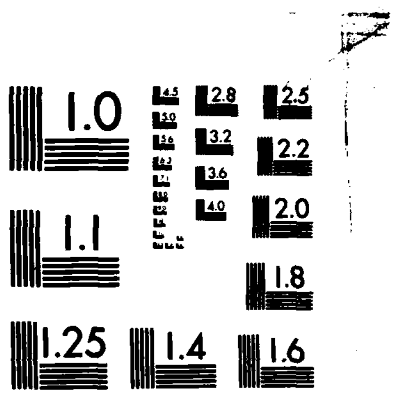

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